Simplified Nonlinear Analysis Procedure for Single-Story Asymmetric Buildings Subjected to Bi-Directional Ground Motion

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Abstract: A simplified procedure for single-story asymmetric buildings subjected to bi-directional ground motion is proposed. In this procedure, their responses are predicted through a nonlinear static analysis of MDOF model considering the effect of bi-directional excitation and a nonlinear dynamic analysis of equivalent SDOF model. The results are compared with those of the nonlinear dynamic analysis of MDOF models, and satisfactory prediction can be found in nonlinear response of asymmetric buildings.

1. INTRODUCTION

The estimation of nonlinear response of buildings subjected to a strong ground motion is a key issue for the rational seismic design of new buildings and the seismic evaluation of existing buildings (ATC-40, 1996). For this purpose, the nonlinear time-history analysis of Multi-Degree-Of-Freedom (MDOF) model might be one solution, but it is often too complicated whereas the results are not necessarily more reliable due to uncertainties involved in input data. To overcome such shortcomings, several researchers have developed simplified nonlinear analysis procedures (Saiidi and Sozen 1981, Fajfar and Fischinger 1988). This approach consists of a nonlinear static (pushover) analysis of MDOF model and a nonlinear dynamic analysis of the equivalent Single-Degree-Of-Freedom (SDOF) model, and it would be a promising candidate as long as buildings oscillate predominantly in the first mode. Although these procedures have been more often applied to planar frame analyses, only a few investigations concerning the extension of the simplified procedure to asymmetric buildings have been made.

In this paper, a simplified procedure for single-story asymmetric buildings (one mass three degree of freedom model) subjected to bi-directional ground motion is proposed. The procedure proposed in this paper is aimed to extend the studies by the authors (Fujii et al. 2003). It consists of a pushover analysis of MDOF model and a nonlinear dynamic analysis of equivalent SDOF model as is in the previous studies (Fajfar et al. 2002), but the effect of bi-directional excitations is taken into account in the pushover analysis. The results obtained by the proposed procedure are compared with those obtained by the nonlinear dynamic analysis of MDOF models. Since the simplified nonlinear analysis procedure under unidirectional excitation proposed in the previous study (Fujii et al. 2003) is applicable only to torsionally stiff (TS) buildings (Fajfar et al. 2002, Fujii et al. 2003), the discussion in this paper is also limited to TS buildings. This discussion made in this paper is the basis to predict the earthquake response of multi-story asymmetric building with simplified procedure, and the applicability of the proposed procedure to multi-story asymmetric buildings will be discussed elsewhere.

2. BUILDING AND GROUND MOTION DATA

2.1 Building Data

Buildings investigated in this paper are idealized single-story asymmetric buildings (one mass three degree of freedom model): they are assumed to be symmetric about the X-axis as shown in Figure 1. Their height is assumed 10.8m and the total building weight is 21.2 MN and the weight is uniformly distributed. In this study, four analytical models are studied considering following parameters: **(1) type of structural plan**, **(2) yield strength in X and Y-direction**.

(1) Type of structural plan: Two structural plans are studied as shown in Figure 1. Both models are symmetric about X-axis and asymmetric about Y-axis. Figure 2 shows the envelope curve of restoring force-displacement relationship of each element. The envelopes are assumed symmetric in both positive and negative loading directions. The Takeda hysteretic model (Takeda et al. 1970) is employed for both column and wall elements, assuming that they behave in a ductile manner. For column elements, the effect of bi-axial moment is neglected for the simplicity of the analysis.

(2) Yield strength in X and Y-direction: Two series of the yield strength in X and Y-direction are studied for each structural plan. Table 1 shows the yield strength of each element and model parameters, where E is the eccentricity ratio (= e / r; e: elastic eccentricity, r: radius of gyration of floor), and J is the radius ratio of gyration of story stiffness (= j / r, j: radius of gyration of story stiffness with respect to the center of mass), Re is the eccentricity ratio in accordance with the Japanese Standard of Seismic Design of Buildings.

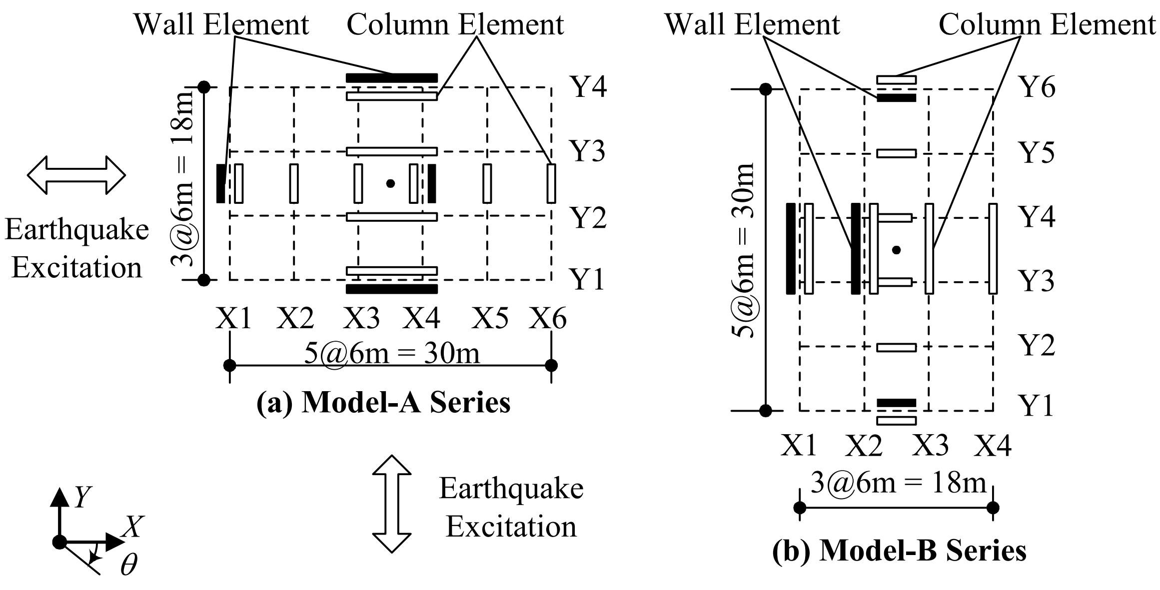


Figure 1 Plan of the Model Building

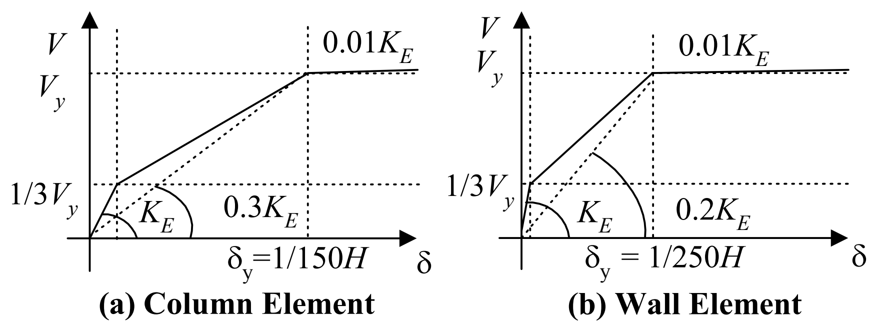
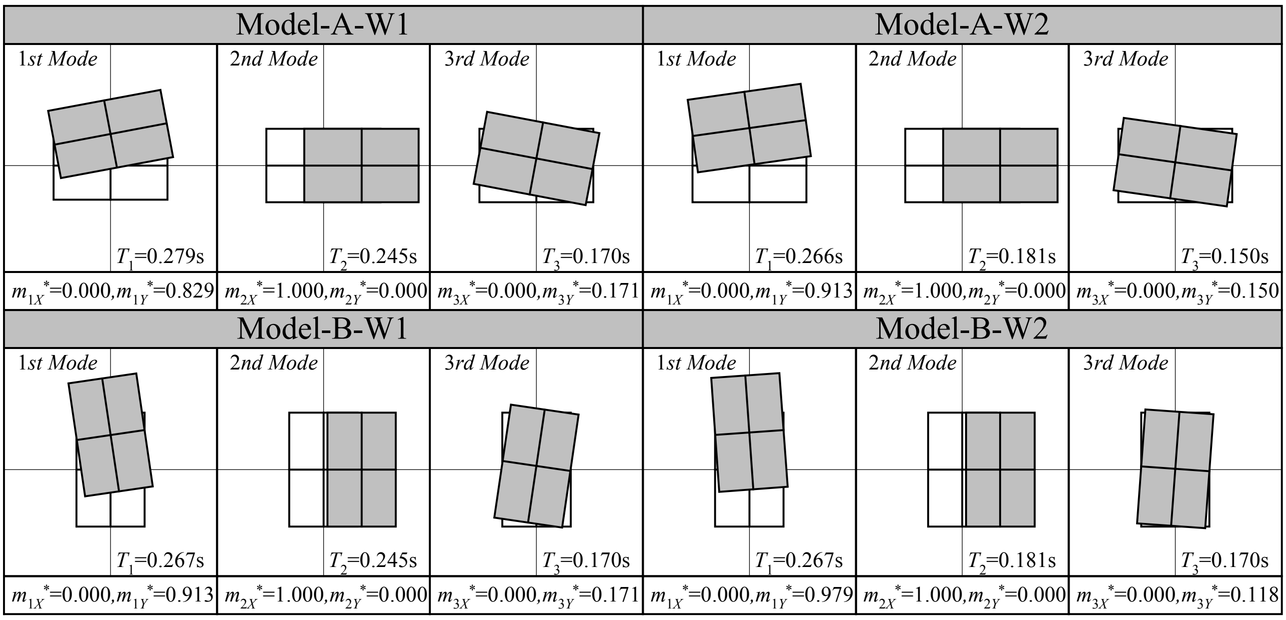


Figure 2 Envelope of Restoring Force-Displacement Relationship

Table 1 Model Parameters

	Yield Strength of Element				E	J	Re
	X-direction		Y-Direction				
	Column	Wall	Column	Wall			
Model-A-W1	$0.06W$	$0.24W$	$0.04W$	$0.24W$	0.495	1.365	0.389
Model-A-W2		$0.48W$				1.589	0.328
Model-B-W1	$0.04W$	$0.24W$	$0.06W$			1.566	0.333
Model-B-W2		$0.48W$				2.071	0.246



Where m_{iX}^*, m_{iY}^* are the equivalent modal mass ratio of i-th mode in X- and Y-axis, respectively (see Ch. 3).

Figure 3 Mode Shapes of Model Buildings

Model-W1 Series: The yield strengths in X and Y-direction are assumed $0.72W$. The total yield strengths of column and wall elements are assumed $0.24W$, $0.48W$, respectively, in each direction.

Model-W2 Series: The yield strengths of the wall elements in X-direction are assumed twice of that in Y-direction. Therefore, the yield strength of those models is $1.20W$in X-direction and $0.72W$ in Y-direction, respectively.

Figure 3 shows the mode shapes of each model. As shown in this figure, the first and second modes of all models are governed by translational component in Y- and X-direction, respectively, while their third modes is governed by the torsional component. Consequently, all models can be classified as torsionally stiff (TS) buildings (Fajfar et al. 2002, Fujii et al. 2003).

2.2 Ground Motion Data

In this study, the earthquake excitation is considered bi-directional in X-Y plane, and six sets of artificial ground motions are used. Target elastic spectrum with 5% of critical damping $S_A(T, 0.05)$ is determined by Eq. 1:

$$S_A(T, 0.05) = \begin{cases} 4.8 + 45T \quad \text{m/s}^2 & T < 0.16\text{s} \\ 12.0 & 0.16\text{s} \le T < 0.864\text{s} \\ 12.0 \cdot (0.864/T) & T \ge 0.864\text{s} \end{cases} \tag{1}$$

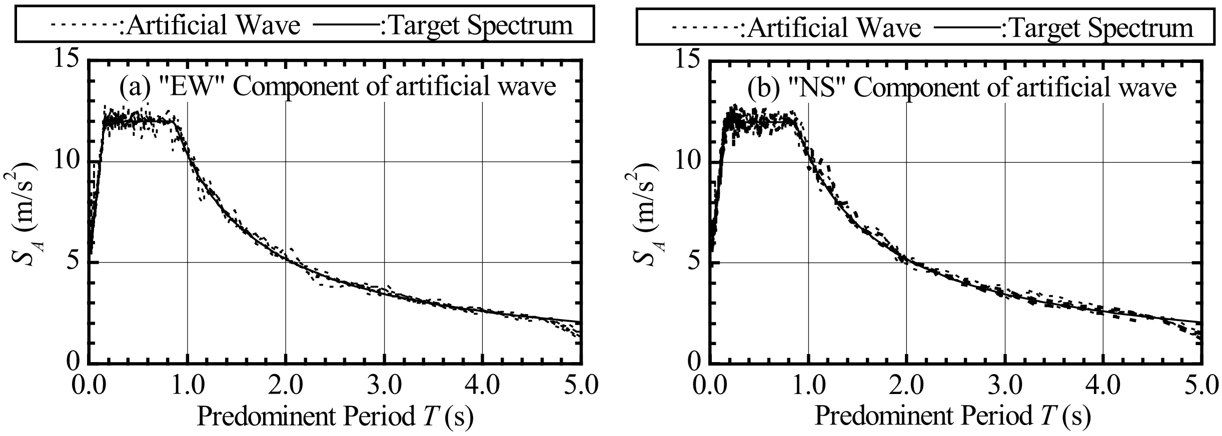


Figure 4 Acceleration Response Spectra

where T is the natural period of the SDOF model. The first 40.96 seconds (2^{12} = 4096 data, 0.01 second sampling) of EW and NS components of the following records are used to determine phase angles of the ground motion: El Centro 1940 (referred to as ELC), Taft 1952 (TAF), Hachinohe 1968 (HAC), Tohoku University 1978 (TOH), Kobe Meteorological Observatory 1995 (JKB) and Fukiai 1995 (FKI). Elastic acceleration response spectra of artificial ground motions with 5% of critical damping are shown in Figure 4. In this study the "EW" and "NS" components of those artificial ground motions are applied simultaneously.

2.3 Numerical Analysis Procedure

In this study, the damping matrix is assumed proportional to the instant stiffness matrix and 3% of the critical damping for the first mode. Newmark-β method (β = 1 / 4) is applied in numerical integrations. The time increment for numerical integration is 0.005 sec. The unbalanced force due to stiffness change is corrected at a next time step during analysis.

3. EQUATIONS OF MOTION OF THE EQUIVALENT SDOF MODELS

The equation of motion of a single-story asymmetric building model can be expressed by Eq. (2):

$$[M]\{\ddot{d}\}+[C]\{\dot{d}\}+\{R\}=-[M](\{\alpha_X\}\cdot a_{gX}+\{\alpha_Y\}\cdot a_{gY}) \tag{2}$$

where $[M]$: mass matrix, $[C]$: damping matrix, $\{d\} = \{x, y, \theta\}^T$: displacement vector (displacement at the C.M. and rotation), $\{R\} = \{V_X, V_Y, T_Z\}^T$: restoring force vector (shear forces in X- and Y-direction and torque at the C.M.), m: mass, I: moment of inertia, $\{\alpha_X\} = \{1, 0, 0\}^T$, $\{\alpha_Y\} = \{0, 1, 0\}^T$: vector defing the directions of ground motion, a_{gX}, a_{gY}: ground acceleration in X- and Y-direction, respectively.

The displacement vector $\{d\}$ and restoring force vector $\{R\}$ are assumed in the form of Eq. (3) even if the building responds beyond the elastic range:

$$\{d\}=\sum_{i=1}^{3}\{\phi_i\}\cdot(\Gamma_{iX}D_{iX}^*+\Gamma_{iY}D_{iY}^*),\{R\}=[M]\sum_{i=1}^{3}\{\phi_i\}\cdot(\Gamma_{iX}A_{iX}^*+\Gamma_{iY}A_{iY}^*) \tag{3}$$

$$\Gamma_{iX}=\frac{\{\phi_i\}^T[M]\{\alpha_X\}}{\{\phi_i\}^T[M]\{\phi_i\}},\Gamma_{iY}=\frac{\{\phi_i\}^T[M]\{\alpha_Y\}}{\{\phi_i\}^T[M]\{\phi_i\}} \tag{4}$$

where Γ_{iX}, Γ_{iY}: i-th modal participation factor, $\{\phi_i\}$: i-th mode shape vector, D_{iX}^*, D_{iY}^*: i-th mode equivalent displacement, A_{iX}^*, A_{iY}^*: i-th mode equivalent acceleration.

It is assumed that a building oscillates predominantly in the first mode under Y-directional (unidirectional) excitation, and under X-directional excitation it oscillates predominantly in the second mode. Eq. (3) is rewritten as Eq. (5), assuming the predominant oscillation of the fundamental modes in both X- and Y-directions under bi-directional excitation and neglecting minor modal responses.

$$\{d\} = \Gamma_{2X}\{\phi_2\}\cdot D_{2X}^* + \Gamma_{1Y}\{\phi_1\}\cdot D_{1Y}^*, \{R\} = [M]\left(\Gamma_{2X}\{\phi_2\}\cdot A_{2X}^* + \Gamma_{1Y}\{\phi_1\}\cdot A_{1Y}^*\right) \tag{5}$$

By substituting Eq. (5) into Eq. (2) and by multiplying $\Gamma_{1Y}\{\phi_1\}^T$ from the left side, Eq. (6) is obtained:

$$\ddot{D}_{1Y}^* + \frac{C_{1Y}^*}{M_{1Y}^*}\cdot \dot{D}_{1Y}^* + A_{1Y}^* = -\left(\sqrt{\frac{m_{1X}^*}{m_{1Y}^*}}\cdot a_{gX} + a_{gY}\right) \tag{6}$$

$$m_{1X}^* = M_{1X}^*/m, m_{1Y}^* = M_{1Y}^*/m \tag{7}$$

$$M_{1X}^* = \Gamma_{1X}^2\left(\{\phi_1\}^T[M]\{\phi_1\}\right) = \Gamma_{1X}\{\phi_1\}^T[M]\{\alpha_X\}, M_{1Y}^* = \Gamma_{1Y}^2\left(\{\phi_1\}^T[M]\{\phi_1\}\right) = \Gamma_{1Y}\{\phi_1\}^T[M]\{\alpha_Y\} \tag{8}$$

$$C_{1Y}^* = \Gamma_{1Y}^2\left(\{\phi_1\}^T[C]\{\phi_1\}\right) \tag{9}$$

where m_{1X}^*, m_{1Y}^* are first equivalent modal mass ratio in X- and Y-axis, respectively, and C_{1Y}^* is first equivalent damping coefficient in Y-axis. In the Eq. (6), m_{1X}^* is zero in elastic range if the building considered is symmetric about X axis as shown in Figure 3. Therefore, assuming that m_{1X}^* is negligibly small even building responds beyond the elastic range, Eq.(6) can be rewritten as Eq.(10).

$$\ddot{D}_{1Y}^* + \frac{C_{1Y}^*}{M_{1Y}^*}\cdot \dot{D}_{1Y}^* + A_{1Y}^* = -a_{gY} \tag{10}$$

Eq. (11) is obtained similarly by substituting Eq. (5) into Eq. (2) and by multiplying $\Gamma_{2X}\{\phi_2\}^T$ from the left side.

$$\ddot{D}_{2X}^* + \frac{C_{2X}^*}{M_{2X}^*}\cdot \dot{D}_{2X}^* + A_{2X}^* = -a_{gX} \tag{11}$$

Eqs. (10) and (11) are the equations of motion of equivalent SDOF models and they are the same as the equation of motion of equivalent SDOF models under unidirectional excitation (Fujii et al. 2003). It should be pointed out that it is rigorous in case of unidirectional excitation, however in case of bi-directional excitation, Eqs. (10) and (11) are approximate: in these equations, the influence of the orthogonal ground motions to response of equivalent SDOF model are neglected by assuming m_{1X}^* and m_{2Y}^* are negligibly small.

4. DESCRIPTION OF SIMPLIFIED NONLINEAR ANALYSIS PROCEDURE

In this chapter, a simplified nonlinear analysis procedure for single-story asymmetric buildings subjected to bi-directional ground motion is proposed. The outline of the proposed procedure is described as follows.

STEP 1: Pushover analysis of MDOF model
STEP 2: Determination of equivalent SDOF model properties
STEP 3: Estimation of seismic demand of equivalent SDOF model
STEP 4: Estimation of drift demand in each frame of MDOF model

The procedure required in each step is described below.

STEP 1: Pushover analysis of MDOF model: Pushover analysis of a MDOF model is carried out to obtain the force – displacement relationship, considering the change in the mode shape at each nonlinear stage. The pushover analysis is carried out independently in both X- and Y-directions (first and second modes), respectively. The numerical procedure of the pushover analysis can be found in previous work by the authors (Fujii et al. 2003).

STEP 2: Determination of equivalent SDOF model properties: The properties of two equivalent SDOF models representing the first and second mode responses are determined from the results of STEP 1, as is in the previous work by the authors (Fujii et al. 2003). For building of which the first and second modes are governed by the translational component in Y- and X-direction, respectively, the equivalent acceleration A_{1Y}^* (or A_{2X}^*) - equivalent displacement D_{1Y}^* (or D_{2X}^*) relationship of the equivalent SDOF models are determined from pushover analysis in STEP 1. A_{1Y}^*, A_{2X}^* and D_{1Y}^*, D_{2X}^* are determined by the Eqs. (12) and (13), respectively:

$$D_{1Y}^* = \frac{\{d_1\}^T[M]\{d_1\}}{\{d_1\}^T[M]\{\alpha_Y\}}, D_{2X}^* = \frac{\{d_2\}^T[M]\{d_2\}}{\{d_2\}^T[M]\{\alpha_X\}} \tag{12}$$

$$A_{1Y}^* = \frac{\{d_1\}^T \cdot \{R_1\}}{\{d_1\}^T[M]\{\alpha_Y\}}, A_{2X}^* = \frac{\{d_2\}^T \cdot \{R_2\}}{\{d_2\}^T[M]\{\alpha_X\}} \tag{13}$$

where $\{d_1\}$, $\{d_2\}$ and $\{R_1\}$, $\{R_2\}$ are the displacement and restoring force vector obtained by the pushover analyses in STEP 1.

The A_{1Y}^*-D_{1Y}^* and A_{2X}^*-D_{2X}^* relationships, referred as to capacity diagram, are idealized by elasto-plastic bi-linear curve so that the hysteretic dissipation enclosed by the original curve and the bi-linear idealized curve is same.

STEP 3: Estimation of seismic demand of equivalent SDOF model: The seismic demand of two equivalent SDOF models $D_{1Y}^*{}_{MAX}$, $D_{2X}^*{}_{MAX}$, $A_{1Y}^*{}_{MAX}$, $A_{2X}^*{}_{MAX}$ are obtained by the equivalent linearization procedure (Otani 2000) in this study. The equivalent period T_{eq} and damping ratio h_{eq} of the equivalent SDOF model at each nonlinear stage is calculated by Eq. (14).

$$T_{eq} = 2\pi\sqrt{D^*/A^*}, h_{eq} = 0.25\left(1 - 1/\sqrt{\mu}\right) + h_0 = 0.25\left(1 - \sqrt{D_Y^*/D^*}\right) + h_0 \tag{14}$$

where μ is the ductility of the equivalent SDOF model, D_Y^* is the yield displacement of the equivalent SDOF model determined from bi-linear curve, and h_0 is the initial damping ratio. In this study, h_0 is assumed 0.03, because in the dynamic analysis of MDOF model the damping is assumed also 3% of critical damping for the first mode as described in section 2.3. The response spectral acceleration and displacement are reduced by following factor F_h calculated by Eq. (15) (Otani, 2000).

$$F_h = 1.5/\left(1 + 10h_{eq}\right) \tag{15}$$

The demand spectrum of an earthquake excitation is constructed by plotting an SDOF response acceleration $S_A(T_{eq}, h_{eq})$ in vertical axis and corresponding displacement $S_D(T_{eq}, h_{eq})$ in the horizontal axis. The seismic demand of equivalent SDOF model is determined by comparing the capacity diagram and the demand spectrum. The intersection of the capacity diagram and demand spectrum represents the maximum response of the equivalent SDOF model.

STEP 4: Estimation of drift demand in each frame of MDOF model: The drift demand in each frame of the MDOF model is determined from the four pushover analyses summarized below.

1) Determination of the four combined force distributions $\{P_{1X}\}$, $\{P_{1Y}\}$, $\{P_{2X}\}$, $\{P_{2Y}\}$ from Eq. (16):

$$\begin{aligned}
\{P_{1X}\} &= [M]\left(\Gamma_{2Xie}\{\phi_{2ie}\}\cdot A_{2X}{}^{*}{}_{MAX} + \gamma\cdot\Gamma_{1Yie}\{\phi_{1ie}\}\cdot A_{1Y}{}^{*}{}_{MAX}\right)\\
\{P_{2X}\} &= [M]\left(\{\alpha_X\}\cdot A_{2X}{}^{*}{}_{MAX} + \gamma\cdot\Gamma_{1Yie}\{\phi_{1ie}\}\cdot A_{1Y}{}^{*}{}_{MAX}\right)\\
\{P_{1Y}\} &= [M]\left(\gamma\cdot\Gamma_{2Xie}\{\phi_{2ie}\}\cdot A_{2X}{}^{*}{}_{MAX} + \Gamma_{1Yie}\{\phi_{1ie}\}\cdot A_{1Y}{}^{*}{}_{MAX}\right)\\
\{P_{1X}\} &= [M]\left(\gamma\cdot\Gamma_{2Xie}\{\phi_{2ie}\}\cdot A_{2X}{}^{*}{}_{MAX} + \{\alpha_Y\}\cdot A_{1Y}{}^{*}{}_{MAX}\right)
\end{aligned} \quad (16)$$

where $\Gamma_{1Yie}\{\phi_{1ie}\}$: first mode shape at $D_{1Y}{}^{*}{}_{MAX}$, $\Gamma_{2Xie}\{\phi_{2ie}\}$: second mode shape at $D_{2X}{}^{*}{}_{MAX}$, and γ: coefficient considering the combination of the first and second.

2) Pushover analysis using $\{P_{1X}\}$ and $\{P_{2X}\}$ until the equivalent displacement D^* calculated by Eq. (17) reaches $D_{2X}{}^{*}{}_{MAX}$ obtained from STEP 3 (referred to as Pushover-1X, Pushover-2X, respectively).

$$D^* = \frac{\Gamma_{2Xie}\{\phi_{2ie}\}^T[M]\{d\}}{\Gamma_{2Xie}\{\phi_{2ie}\}^T[M]\{\alpha_X\}} \quad (17)$$

3) Pushover analysis using $\{P_{1Y}\}$ and $\{P_{2Y}\}$ until the equivalent displacement D^* calculated by Eq. (18) reaches $D_{1Y}{}^{*}{}_{MAX}$ obtained from STEP 3 (Pushover-1Y, Pushover-2Y, respectively).

$$D^* = \frac{\Gamma_{1Yie}\{\phi_{1ie}\}^T[M]\{d\}}{\Gamma_{1Yie}\{\phi_{1ie}\}^T[M]\{\alpha_Y\}} \quad (18)$$

4) Determination of the drift demand by the envelope of (a) Pushover-1X and 2X obtained from 2) and (b) Pushover-1Y and 2Y obtained from 3).

The value of γ is a key parameter to predict the drift at each frame. If $A_{2X}{}^{*}$ equals to zero when $A_{1Y}{}^{*}{}_{MAX}$ occurs, γ is taken as 0.0, while if $A_{1Y}{}^{*}{}_{MAX}$ and $A_{2X}{}^{*}{}_{MAX}$ occurs simultaneously, γ is taken as 1.0. In this study, γ = 0.0, 0.5 and 1.0 are studied.

5. ANALYSIS RESULTS

Figure 5 shows the comparisons of the maximum drift at each frame obtained from time-history analysis of MDOF models (mean value of the 12 analyses, and mean $\pm$ standard deviation are shown) and the proposed procedure. This figure shows that in case of γ = 0.5, the proposed procedure can estimate the drift at each frame satisfactory. However, in case of γ = 0.0, the torsional response is underestimated and therefore the drift at stiff side (Frame X1) is overestimated and the drift at frame Y1 are underestimated, while in case of γ = 1.0, the torsional response is overestimated and therefore the drift at stiff side is underestimated and the drift at flexible side (frame X6 for Model-A series, and frame X4 for Model-B series, respectively) and frame Y1 are significantly overestimated. Therefore the proposed procedure with γ = 0.5 provides the most reasonable predictions in three cases.

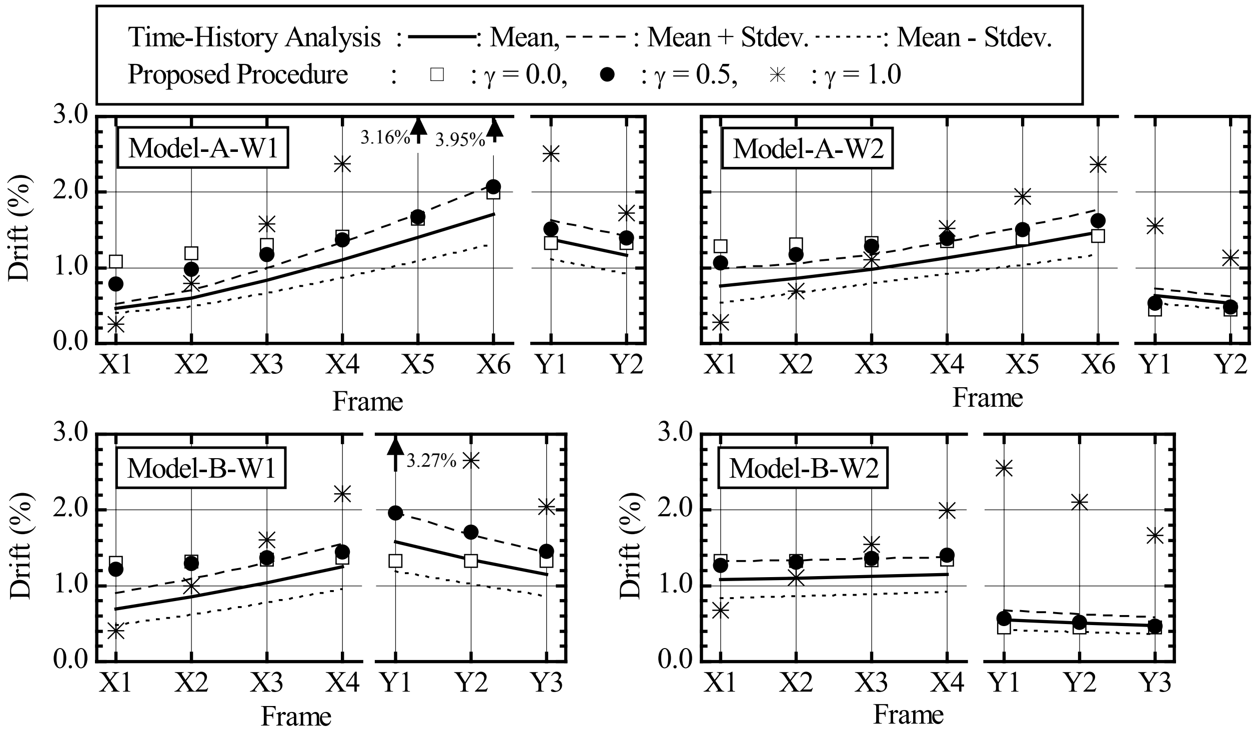


Figure 5 Prediction of the Maximum Drift at Each Frame

6. CONCLUSIONS

In this paper, a simplified procedure for single-story asymmetric buildings subjected to bi-directional ground motion is proposed, and the results obtained by the proposed procedure are compared with the results obtained by the nonlinear dynamic analysis. The results show that the nonlinear response of asymmetric buildings subjected to bi-directional ground motion can be satisfactorily estimated by the simplified procedure proposed in this study.

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