An Evaluation of the Displacement Amplification Factors for Seismic Design of Bridges

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Abstract: This paper presents an analysis of the displacement amplification factors in seismic design of bridges. The displacement amplification factors are evaluated based on 70 free-field ground motions. Scattering of the displacement amplification factors depending on ground motions and natural periods are clarified. A new empirical formulation of the displacement amplification factors is proposed.

1. INTRODUCTION

In the force based seismic design, the force demand is generally determined based on a linear response of a structure divided by the force reduction factor. In the similar way, the displacement demand of a structure is estimated by multiplying a linear displacement response by the displacement amplification factor (Ye and Otani 1999, Miranda and Ruiz-Garcia 2002, 2003, and Lin, Chang and Wang 2004). The force reduction factors are worldwide used, and it plays an important role in the force based design of a structure. However, in spite of the importance of the displacement demand in seismic design of bridges, less attention has been paid to the estimation of a displacement amplification factor.

This paper presents an analysis on the displacement amplification factors based on 70 free-field ground motions. The dependence of displacement amplification factors on natural periods and ground motions is clarified.

2. DEFINITION OF DISPLACEMENT AMPLIFICATION FACTOR

If one idealizes a structure in terms of a single-degree-of-freedom (SDOF) oscillator with an elastic perfect plastic bilinear hysteretic behavior as shown in Fig. 1, the displacement amplification factor D_μ may be defined as

$$D_\mu = \frac{\delta_{NL}(T, \mu_T, \xi_{NL})}{\delta_{EL}(T, \xi_{EL})} \tag{1}$$

where T : natural period, δ_{EL} and δ_{NL} : maximum displacement in an oscillator with a linear and a bilinear hysteresis, respectively, μ_T : target ductility factor, and ξ_{EL} and ξ_{NL} : damping ratio assumed in the evaluation of linear and bilinear responses, respectively. The natural period T may be

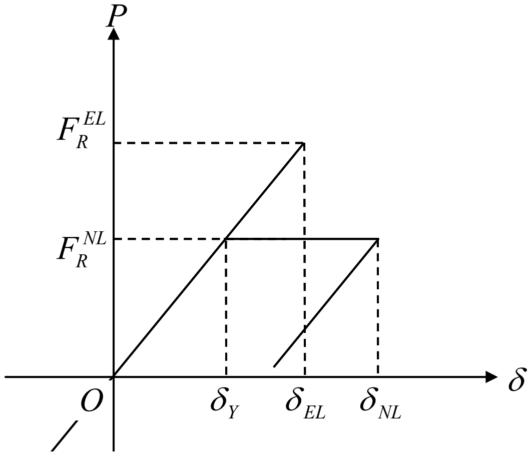


Fig. 1 Linear or Nonlinear Response of an SDOF oscillator

evaluated based on the initial elastic stiffness of columns. Representing u_y the yield displacement where the stiffness changes from the initial elastic stiffness to the post-yield stiffness, a target ductility factor μ_T may be defined based on the yield displacement u_y as

$$\mu_T = \frac{u_{\max}}{u_y} \tag{2}$$

where $u_{\max}$ is the maximum displacement response of an oscillator. The post-yield stiffness is assumed to be 0 in the present study.

An important point in Eq.(1) is what value should be assigned to ξ_{EL} and ξ_{NL}. It is general to assume ξ_{EL} = 0.05, which may be validated from the fact that standard structures have a damping ratio of nearly 0.05. On the other hand, because an energy dissipation occurs in the bilinear oscillator resulted from the hysteretic response, assuming ξ_{NL} = 0.05 results in larger energy dissipation in the bilinear oscillators than the linear oscillator. Because taking account of the hysteretic energy dissipation, it is often the practice to assume a damping ratio which is slightly smaller than ξ_{EL} in the nonlinear always, ξ_{NL} is assumed 0.02 here.

3. DISPLACEMENT AMPLIFICATION FACTOR FOR BILINEAR OSCILATORS

Displacement amplification factors were evaluated at the target ductility factor μ_T of 2, 4, 6 and 8 assuming an elastic perfected-plastic bilinear hysteresis. Seventy free-field ground accelerations by 64 shallow earthquakes with depth less that 60 km were used in analysis. They were classified into three soil conditions depending on the fundamental natural period of subsurface ground T_g; stiff (T_g < 0.6 s), moderate (0.2 < T_g < 0.6 s) and soft (0.6 s < T_g) (Japan Road Association 2002). Number of records in the stiff, moderate and soft categories is 16, 39 and 15, respectively. Distribution of peak ground accelerations on the earthquake magnitudes and epicentral distances is shown in Fig. 2. The peak accelerations are in the range of 0.1-8 m/sec^2, and the epicentral distances are in the range of 10-500 km.

Fig. 3 shows the displacement amplification factors for the 70 ground motions. Only the results for μ_T = 4 and 6 are presented here since the results for other target ductility factors show the similar characteristics. It is seen in Fig. 3 that scattering of the displacement amplification factors depending on ground motions is considerable. For example, the displacement amplification factors at natural

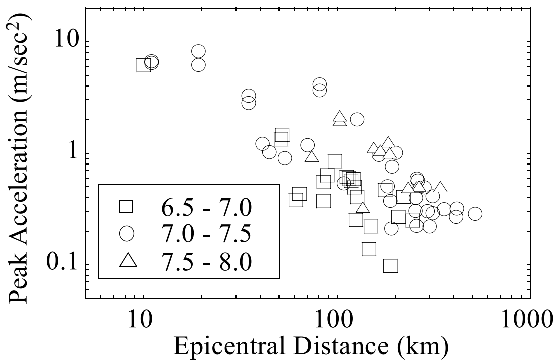


Fig. 2 Classification of Ground Acceleration in terms of Earthquake Magnitudes

period of 1 second varies from 0.29 to 3.41 depending on ground motions at $\mu_T = 4$. The dependence of displacement amplification factors on the soil condition is less significant.

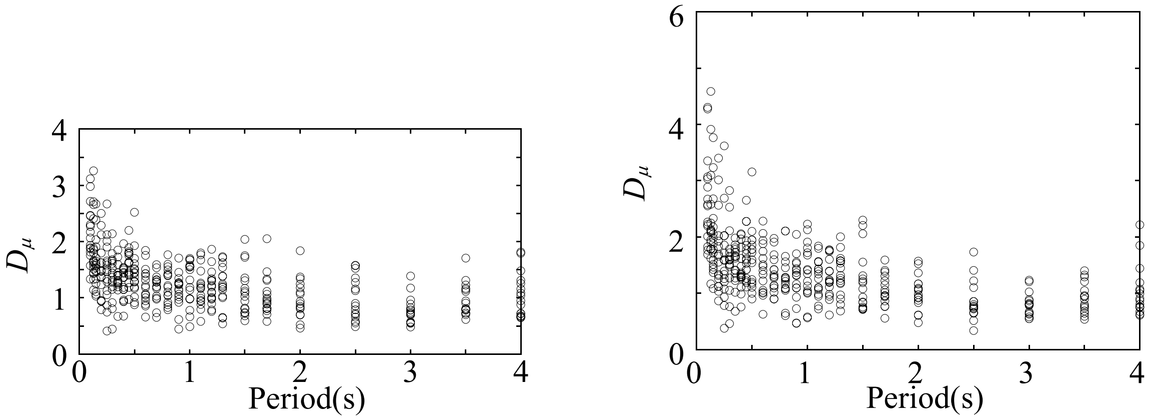
Since the scattering of the displacement amplification factors depending on ground motions is considerable, the averages +/- one standard deviations of the displacement amplification factors were obtained at each target ductility factor, natural period and soil condition. Fig. 4 shows the average values and the average values +/- one standard deviations of the displacement amplification factors at $\mu_T = 4$. The following displacement amplification factors predicted by the equal displacement and the equal energy assumptions are presented here for comparison.

$$D_\mu = \begin{cases} 1 & \text{Equal Displacement Assumption} \\ \mu/\sqrt{2\mu - 1} & \text{Equal Energy Assumption} \end{cases} \tag{3}$$

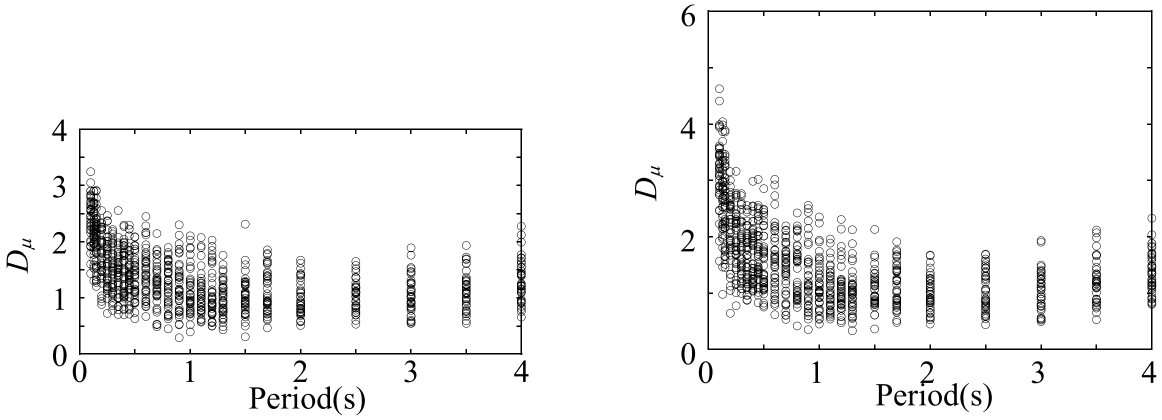
The average values of displacement amplification factors sharply increase as the natural periods decrease, while they approach to 1.0 at the periods longer than about 1.5s. The equal displacement assumption provides a good estimate to the average values at the natural periods longer than about 1.5 s. But if we take account of the considerable scattering in terms of the average plus one standard deviation, the equal displacement assumption considerably underestimates the displacement amplification factors. On the other hand, the equal energy assumption provides an overestimation to the average values but better estimation to the average values plus one standard deviations. Taking the considerable scattering of the displacement amplification factors depending on ground motions into account, it seems reasonable to consider a certain redundancy in the estimate of the displacement amplification factors in design. For such a purpose, it seems appropriate to assume the equal energy assumption instead of the equal displacement assumption.

Fig. 5 shows the dependence of the standard deviations of displacement amplification factors $\sigma(D_\mu)$ on the natural periods T and the soil condition. Different to the average values, the standard deviations $\sigma(D_\mu)$ are less dependent on the natural periods T. Fig. 6 shows the dependence of the standard deviations $\sigma(D_\mu)$ on the target ductility factors μ_T. The standard deviations $\sigma(D_\mu)$ increase as the target ductility factors increase. The relation may be approximated by a least square fit as

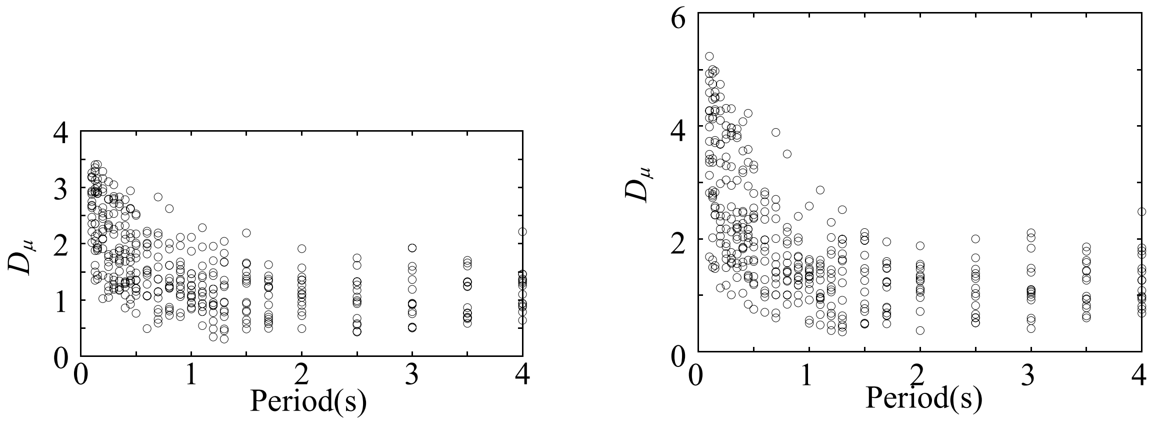
$$\sigma(D_\mu) = \begin{cases} 0.145 + 0.053 \cdot \mu_T & (Stiff) \\ 0.170 + 0.049 \cdot \mu_T & (Medium) \\ 0.061 + 0.101 \cdot \mu_T & (Soft) \end{cases} \tag{4}$$



(a) Stiff (Type-I)



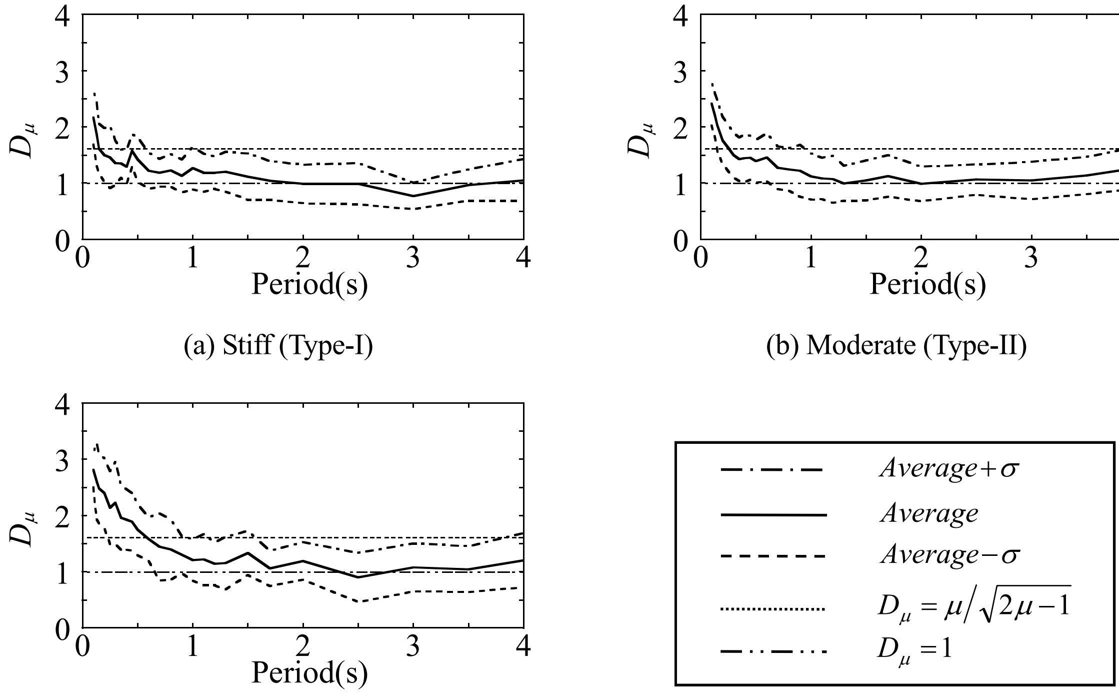
(b) Moderate (Type-II)



(c) Soft (Type-III)

(1) $\mu_T = 4$ (2) $\mu_T = 6$

Fig. 3 Displacement Amplification Factor



(a) Stiff (Type-I)

(b) Moderate (Type-II)

(c) Soft (Type-III)

Fig. 4 Average and Average +/- One Standard Deviation of the Displacement Amplification Factor ($\mu_T = 4$)

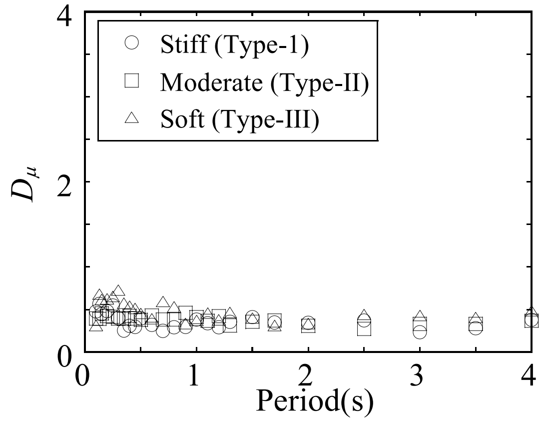


Fig. 5 Natural Period Dependencies of the Standard Deviations of the Displacement Amplification Factors

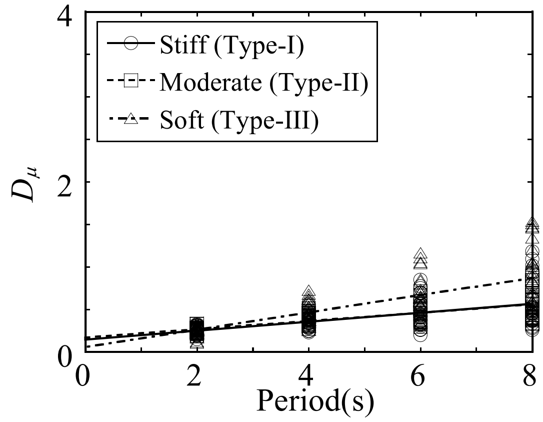


Fig. 6 Target Ductility Factor Dependencies of the Standard Deviations of the Displacement Amplification Factors

4. DISPLACEMENT AMPLIFICATION FACTOR FOR BILINEAR OSCILATORS

To idealize the average values of the displacement amplification factors, they are formulated as

$$D_\mu = \Psi(T) + 1 \tag{5}$$

where,

$$\Psi(T) = -(c-1) \cdot \frac{T-a}{ae^{b \cdot T}} \tag{6}$$

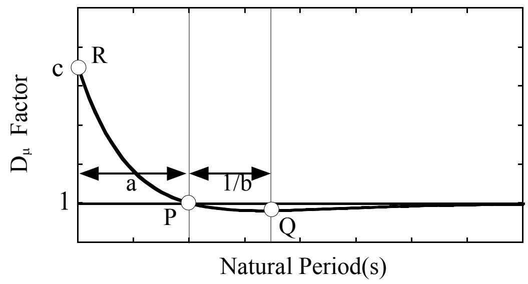


Fig. 7 Idealization of Force Reduction Factors

Table. 1 Parameters a, b and Regression Coefficients R

μ_T	a, b, c and R	Soil Conditions			μ_T	a, b, c and R	Soil Conditions		
		Stiff	Moderate	Soft			Stiff	Moderate	Soft
2	a	1.19	1.37	1.51	4	a	1.34	1.31	1.64
	b	1.50	2.28	1.30		b	1.69	2.42	1.46
	c	1.39	1.74	1.71		c	2.02	2.66	2.96
	R	19.6	46.6	55.1		R	54.1	65.6	70.2
6	a	1.32	1.32	1.68	8	a	1.27	1.36	1.76
	b	1.66	1.89	1.53		b	1.99	1.91	1.73
	c	2.50	3.29	4.19		c	3.17	4.02	5.48
	R	57.9	72.6	72.9		R	59.9	74.7	75.2

in which a, b and c are parameters depending on the natural periods and the soil conditions. Since $D_\mu = \mu$ at $T = a$ and $D_\mu = c$ at $T = 0$ in Eqs. (5) and (6), the parameter a and c represent the period where D_μ is equal to 1 (point P) and the displacement amplification factor D_μ at $T = 0$ as shown in Fig. 7, respectively. Since the gradient of D_μ is

$$\frac{dD_\mu}{dT} = -(c-1) \cdot \frac{1 - b(T-a)}{a \cdot e^{-b \cdot T}} \tag{7}$$

representing Q as the point where D_μ takes the minimum value, $1/b$ represents the period between points P and Q.

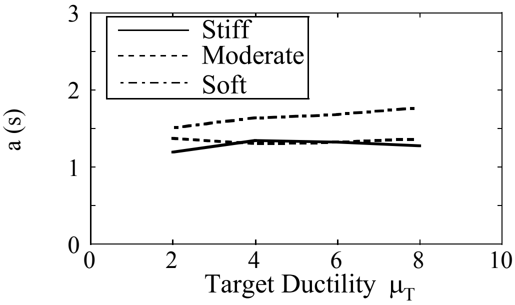
Based on the definition, Eqs. (5) and (6) automatically satisfy the following conditions

$$\lim_{T \to 0} D_\mu = \mu \tag{8}$$

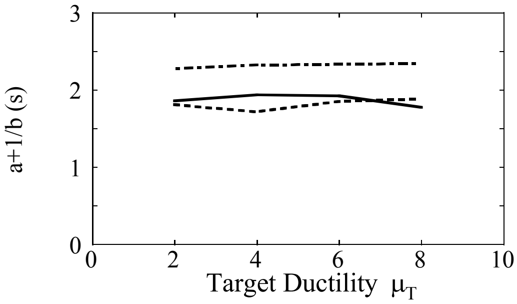
$$\lim_{T \to \infty} D_\mu = 1 \tag{9}$$

The average values of the displacement amplification factors in Fig. 4 were fitted by Eq. (5) using a nonlinear least square method (Press et al 1996). Table 1 represents the parameters a, b and c as well as the regression coefficients R. Since the regression coefficients R are not high enough at some combinations of the target ductility factors and soil conditions, such as the combination of μ_T = 2 and stiff sites, this should be noted in the interpretation of the following results.

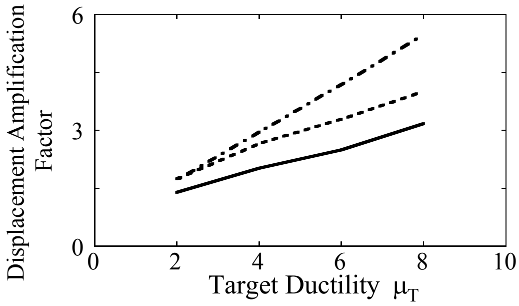
Fig. 8 shows the parameters a, $a + 1/b$ and c. The parameter a is in the range of 1.3-1.4 second at the stiff and the moderate sites, and 1.5-1.8 second at the soft sites. They are less sensitive to



(a) Parameters a



(b) Parameters $a+1/b$



(c) Parameters c

Fig. 8 Parameters a, $a+1/b$ and c in Eq.(6)

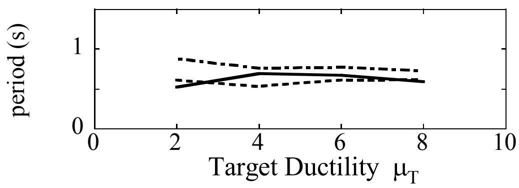


Fig. 9 Natural period where $D_\mu = \mu / \sqrt{2\mu - 1}$

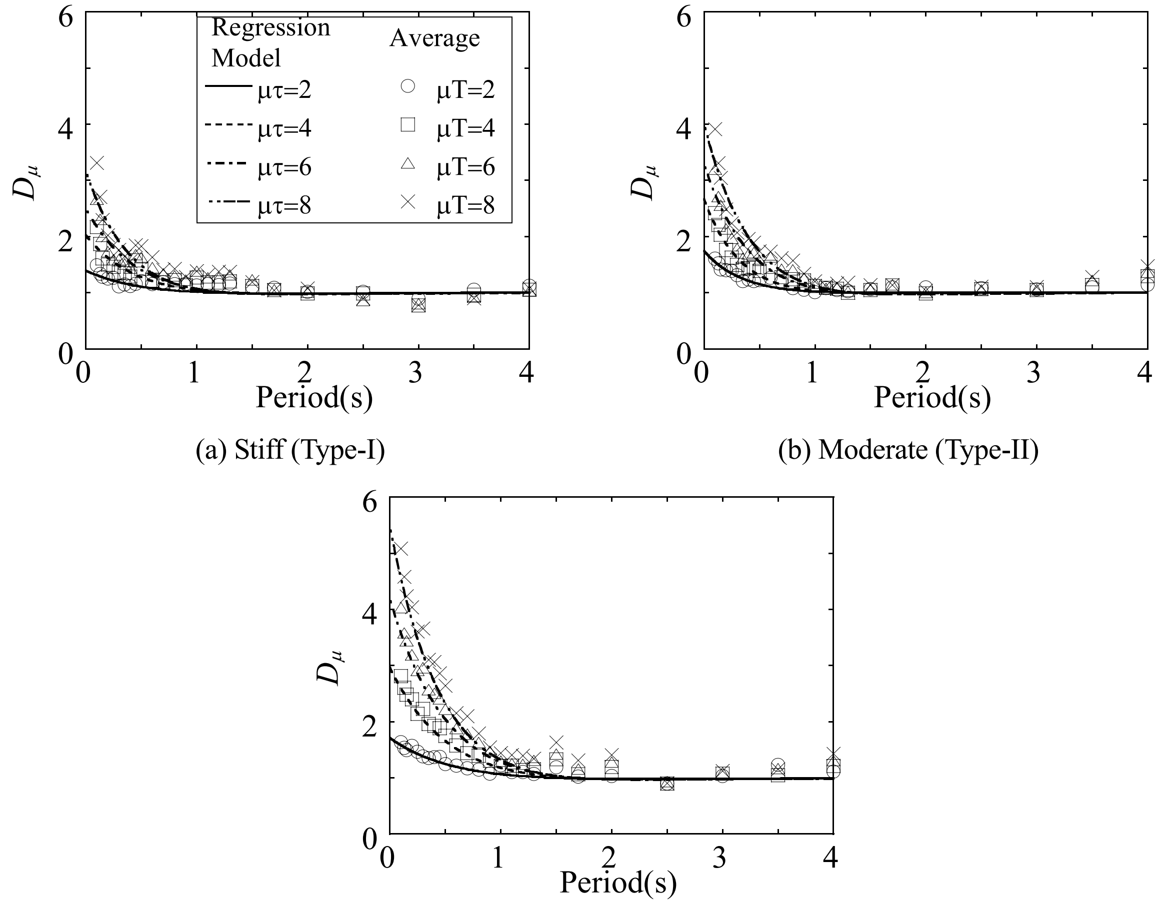
the target ductility factor μ_T. As described above, a represents the period where $D_\mu = 1.0$, which implies that the equal displacement assumption provides the best estimate at this period. Consequently, the accuracy of the equal displacement assumption is high at 1.0-1.4 second at the stiff and the moderate sites, and 1.5-2.4 second at the soft site. On the other hand, $a+1/b$ represents the natural period where D_μ takes the minimum value. It is 1.8-1.9, 1.7-1.9 and 2.3 second at the stiff, moderate and soft sites, respectively. The parameter c is in the range of 1.3-3.2, 1.7-4.0 and 1.7-5.5 at the stiff, moderate and soft sites, respectively. It increases as the target ductility μ_T increases.

The natural periods T where D_μ is equal to $\mu_T / \sqrt{2\mu_T - 1}$ are obtained as shown in Fig. 9, based on Eq.(3), the equal energy assumption provides the best estimate in the range of 0.52-0.69, 0.53-0.62 and 0.72-0.88 at the stiff, the moderate and the soft sites, respectively. They are much shorter than the natural periods where the equal displacement assumption provides the best approximation.

Fig. 10 compares the average displacement amplification factors presented in Fig. 4 and the values predicted by Eqs. (5) and (6). Although slightly discrepancies are observed at larger target ductility factors, Eqs. (5) and (6) provides a good estimation for the average displacement amplification factors.

5. CONCLUSIONS

An analysis was conducted on the displacement amplification factors of SDOF oscillators based on the 70 free-field ground motions. Based on the analysis presented herein, the following conclusions may be deduced:



(a) Stiff (Type-I)

(b) Moderate (Type-II)

(c) Soft (Type-III)

Fig. 10 Displacement Amplification Factor

1) Scattering of the displacement amplification factors depending on the ground motions is considerable.
2) The equal displacement assumption and the equal energy assumptions provide a good estimate to the average and average plus one standard deviation of the displacement amplification factors, respectively. Taking account of the considerable scattering, it seems appropriate to assume the equal energy assumption instead of the equal displacement assumption for the evaluation of the displacement amplification factors in seismic design.
3) An empirical equation to predict the displacement amplification factors was proposed as shown in Eqs. (5) and (6). The parameters a and $a + 1/b$ express the natural period where D_μ is equal to μ and D_μ becomes the minimum, respectively.

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