Beam Element For Truss Beam With Elastoplastic-Buckling Behavior

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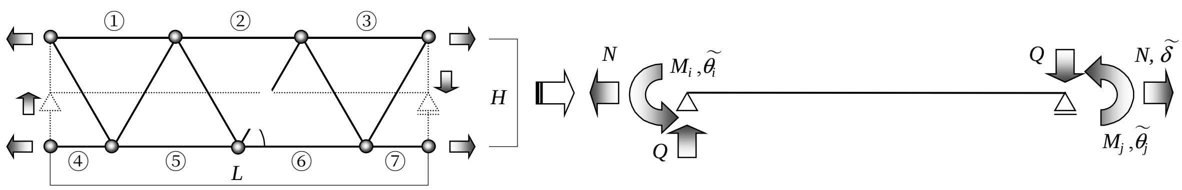
Abstract: In this paper, we will suggest the useful beam element which enables us to analyze truss beams involving the elastoplastic buckling behavior of chord members without constructing discrete model. In this element, multi-surfaces for the yielding and buckling behavior are considered in the space of (M_i, M_j, N). Each couple of surfaces corresponds to the yielding and buckling strength of each chord member in a truss beam. It is assumed that total nodal displacement can be expressed in the form of additive decomposition of elastic, plastic and buckling components. Furthermore, we describe that it is possible to evaluate the effect of buckling behavior of a chord member as the isotropic softening behavior for only a couple of surfaces in the space of (M_i, M_j, N). Finally we will examine the validity of our beam element through a numerical example.

1. INTRODUCTION

We consider a truss beam as shown in Figure 1(a). This truss beam belongs to the Warren truss type. Usually structural designers do not directly analyze such huge structures as are composed with many truss beams since such analytical models have much more freedom degree number and are very costly. Then an effective continuous model in which a truss beam is replaced with a single beam element shown in Figure 1(b), is usually used when analyzing the dynamic behavior of such a huge structure. The continuous model could not applied to the plastic problem until we developed a truss beam element which enabled us to simulate elastoplastic problem (Motoyui et al.(2000b)). However, chord members buckle and can not carry an axial force as soon as they yield in compression. In this paper, we will describe the consistent and convenient truss beam element to consider the elastoplastic buckling behavior of chord members in the continuous model.

At first, we explain that the elastoplastic buckling behavior of a chord member can be approximately represented as the plastic behavior with softening under the assumption that its slenderness ratio is the less(Motoyui et al.(2000a)). Next, we formulate the truss beam element based on the thermodynamics approach. Finally we show the validity of the present element by comparison with results by a discrete model of which chord members are divided by standard beam elements.



(a) Discrete model (b) Effective continuous model

Figure 1 Two types of truss model

2. BASIC EQUATIONS FOR CHORD MEMBER

It is assumed that the Helmholtz free energy ψ_α for the α th chord member which consist of an elastic straight bar and a perfect elastoplastic rotary spring in Figure 2(b) can be written by

$$\psi_\alpha\left(u_\alpha,\theta_\alpha,u_\alpha^p,\theta_\alpha^p\right)=\left\{\frac{1}{2}{}_nk_\alpha\left(\frac{u_\alpha}{2}-\frac{u_\alpha^p}{2}-\frac{l_\alpha}{4}\theta_\alpha{}^2\right)^2+\frac{1}{2}{}_mk_\alpha\left(\theta_\alpha-\theta_\alpha^p\right)^2\right\}\times 2 \tag{1}$$

where u_α and θ_α are a total stretch and total rotation, u_α^p and θ_α^p are plastic components of a stretch and a rotation, and ${}_nk_\alpha$ and ${}_mk_\alpha$ are an elastic stiffness of a bar and an elastic stiffness of a rotary spring. The sub-suffix α means the value of the α-th member. In this paper, it is asumed that θ is always positive and a chord member does not buckle until its member yields.
Substituting in the Clausius-Duhem inequality; $-\dot{\psi}_\alpha+n_\alpha\dot{u}_\alpha\geq 0$ from Eq.(1) gives

$$n_\alpha=\frac{1}{2}{}_nk_\alpha\left(u_\alpha-u_\alpha^p-\frac{l_\alpha}{2}\theta_\alpha{}^2\right),\qquad n_\alpha l_\alpha\theta_\alpha=2{}_mk_\alpha\left(\theta_\alpha-\theta_\alpha^p\right)\Rightarrow 2m_\alpha,\qquad \Gamma_\alpha=n_\alpha\dot{u}_\alpha^p+n_\alpha l_\alpha\theta_\alpha\dot{\theta}_\alpha^p\geq 0 \tag{2}$$

where Γ_α is called the plastic dissipation term and this expression mean variables of n_α and $n_\alpha l_\alpha\theta_\alpha$ are thermodynamic force to u_α^p and θ_α^p respectively. The yield function at the rotary spring; $\Phi_\alpha(n_\alpha,m_\alpha)=(n_\alpha/n_y)^2+|m_\alpha|/m_p-1\leq 0$ can be rewritten in the form

$$\Phi_\alpha\left(n_\alpha,\theta_\alpha\right)=\frac{|n_\alpha|}{n_y}-\bar{\tau}_\alpha\left(\theta_\alpha\right)\leq 0\quad\text{where}\quad \bar{\tau}_\alpha\left(\theta_\alpha\right)=\bar{\sigma}_\alpha\left(\theta_\alpha\right)-\frac{n_y l_\alpha\theta_\alpha}{4m_p},\quad \bar{\sigma}_\alpha\left(\theta_\alpha\right)=\sqrt{1+\left(\frac{n_y l_\alpha\theta_\alpha}{4m_p}\right)^2} \tag{3}$$

Here, the principle of maximum plastic dissipation is introduced to specify the post-buckling behavior. So consider the Lagrangian; $L_\alpha=-\Gamma_\alpha+\dot{\lambda}_\alpha\Phi_\alpha$ where $\dot{\lambda}_\alpha$ is a plastic consistent parameter. Differentiation of the Lagrangian with respect to n_α or θ_α gives

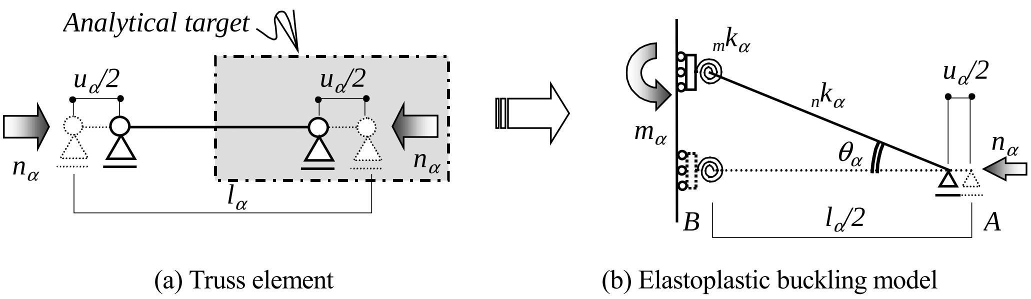
$$\dot{u}_\alpha^p=\frac{n_\alpha}{n_y{}^2}\frac{1}{\bar{\sigma}_\alpha}\dot{\lambda}_\alpha,\qquad \dot{\theta}_\alpha^p=\frac{1}{4m_p}\frac{n_\alpha}{|n_\alpha|}\frac{1}{\bar{\sigma}_\alpha}\dot{\lambda}_\alpha \tag{4}$$

These equations are called evolution equations. Furthermore, if the yield condition of Eq.(3-a) is active then the following Kuhn-Tucker complementary conditions must be satisfied

$$\Phi_\alpha=0,\quad \dot{\Phi}_\alpha=0\qquad\text{and}\qquad \dot{\lambda}_\alpha\geq 0 \tag{5}$$

Now consider the case that member's slenderness ratio is much less. In this case, the assumption that θ_α is approximately equal to θ_α^p is satisfied. Then the yield function of Eq.(3) can be rewritten in the form

$$\Phi_\alpha\left(n_\alpha,\lambda_\alpha^p\right)=\frac{|n_\alpha|}{n_y}-\bar{\tau}_\alpha\left(\lambda_\alpha^p\right)\leq 0\quad\text{where}\quad \bar{\tau}_\alpha\left(\lambda_\alpha^p\right)=\bar{\sigma}_\alpha\left(\lambda_\alpha^p\right)-\frac{n_y l_\alpha\theta_\alpha^p\left(\lambda_\alpha^p\right)}{4m_p},\quad \bar{\sigma}_\alpha\left(\lambda_\alpha^p\right)=\sqrt{1+\left(\frac{n_y l_\alpha\theta_\alpha^p\left(\lambda_\alpha^p\right)}{4m_p}\right)^2} \tag{6}$$



(a) Truss element (b) Elastoplastic buckling model

Figure 2 Single chord member

We introduce an effective plastic-buckling component u_α^{pb} which is defined by the form of additive decomposition.

$$u_\alpha^{pb} = u_\alpha^p + u_\alpha^b, \qquad u_\alpha^b = \frac{l_\alpha}{2}{\theta_\alpha^p}^2, \qquad \dot{u}_\alpha^{pb} = \dot{u}_\alpha^p + l_\alpha \theta_\alpha^p \dot{\theta}_\alpha^p = \frac{1}{n_y}\frac{n_\alpha}{|n_\alpha|}\dot{\lambda}_\alpha \tag{7}$$

From the third loading condition of Eq.(5-c), the necessary condition to satisfy the second law of thermodynamics is given by

$$\frac{1}{{n_y}^2}\frac{{}_n k_\alpha}{2} - \frac{n_\alpha l_\alpha}{16{m_p}^2{\overline{\sigma}_\alpha}^2} > 0 \tag{8}$$

3. EXTEND TO TRUSS BEAM

3.1 Basic Equations for Truss Beam

The Helmholtz free energy Ψ for the whole of a truss beam can be given by the sum of each member's Helmholtz free energy ψ.

$$\Psi = \sum_{\alpha=1}^{all} \psi_\alpha\left(u_\alpha, \theta_\alpha, u_\alpha^p, \theta_\alpha^p\right) \tag{9}$$

This equation can re rewritten in the form

$$\Psi = \frac{1}{2}\tilde{\boldsymbol{u}}^{ef\,T}\boldsymbol{k}^e\tilde{\boldsymbol{u}}^{ef} + \frac{1}{2}\sum_{\alpha=1}^{all}\left\{{}_m k_\alpha\left(\theta_\alpha - \theta_\alpha^p\right)^2 \times 2\right\} \tag{10}$$

where $()^{\mathrm{T}}$ signifies transpose, $\boldsymbol{k}^e$ is an elastic stiffness matrix for an effective continuous model and $\tilde{\boldsymbol{u}}^{ef}$ is the effective elastic component of the nodal relative displacement $\tilde{\boldsymbol{u}}$. And we assume that total component $\tilde{\boldsymbol{u}}$ can be rewritten by the form of additive decomposition.

$$\tilde{\boldsymbol{u}} = \tilde{\boldsymbol{u}}^{ef} + \tilde{\boldsymbol{u}}^p + \tilde{\boldsymbol{u}}^b, \quad \tilde{\boldsymbol{u}}^T = \left\langle \tilde{\delta} \quad \tilde{\theta}_i \quad \tilde{\theta}_j \right\rangle \tag{11}$$

where $\tilde{\boldsymbol{u}}^p$ and $\tilde{\boldsymbol{u}}^b$ are the plastic and buckling components of the nodal relative displacement. Furthermore, the assumption of $\tilde{\boldsymbol{u}}^p \equiv \tilde{\boldsymbol{u}}^p\left(u_\alpha^p\right)$ and $\tilde{\boldsymbol{u}}^b \equiv \tilde{\boldsymbol{u}}^b\left(\theta_\alpha\right)$ is introduced.

$$\dot{\tilde{\boldsymbol{u}}}^p\left(u_\alpha^p\right) = \sum_{\alpha=1}^{all}\frac{\partial \tilde{\boldsymbol{u}}^p}{\partial u_\alpha^p}\dot{u}_\alpha^p \equiv \sum_{\alpha=1}^{all}\boldsymbol{h}_\alpha^p \dot{u}_\alpha^p, \qquad \dot{\tilde{\boldsymbol{u}}}^b\left(\theta_\alpha\right) = \sum_{\alpha=1}^{all}\frac{\partial \tilde{\boldsymbol{u}}^b}{\partial \theta_\alpha}\dot{\theta}_\alpha \equiv \sum_{\alpha=1}^{all}\boldsymbol{h}_\alpha^b \dot{\theta}_\alpha \tag{12}$$

In this case, the Clausius-Duhem inequality can be rewritten as $-\dot{\Psi} + \boldsymbol{f}^T\dot{\tilde{\boldsymbol{u}}} \geq 0$. $\boldsymbol{f}$ is the nodal forces; $\boldsymbol{f}^T = \left\langle N \quad M_i \quad M_j \right\rangle$. Then substituting in this inequality from Eq.(10) gives

$$\left\{\boldsymbol{f} - \boldsymbol{k}^e\tilde{\boldsymbol{u}}^{ef}\right\}^T\dot{\tilde{\boldsymbol{u}}}^{ef} + \sum_{\alpha=1}^{all}\left[\boldsymbol{f}^T\boldsymbol{h}_\alpha^b - 2\,{}_m k_\alpha\left(\theta_\alpha - \theta_\alpha^p\right)\right]\dot{\theta}_\alpha + \sum_{\alpha=1}^{all}\left[\boldsymbol{f}^T\boldsymbol{h}_\alpha^p\dot{u}_\alpha^p + 2\,{}_m k_\alpha\left(\theta_\alpha - \theta_\alpha^p\right)\dot{\theta}_\alpha^p\right] \geq 0 \tag{13}$$

For this inequality equation to be true for all values of $\tilde{\boldsymbol{u}}^{ef}$ or θ_α, their coefficients must be zero, giving

$$\boldsymbol{f} = \boldsymbol{k}^e\tilde{\boldsymbol{u}}^{ef}, \qquad \boldsymbol{f}^T\boldsymbol{h}_\alpha^b = 2\,{}_m k_\alpha\left(\theta_\alpha - \theta_\alpha^p\right), \qquad \sum_{\alpha=1}^{all}\left[\boldsymbol{f}^T\boldsymbol{h}_\alpha^p\dot{u}_\alpha^p + \boldsymbol{f}^T\boldsymbol{h}_\alpha^b\dot{\theta}_\alpha^p\right] \geq 0 \tag{14}$$

The third equation represents the dissipation term. By comparison with Eq.(2-c), it is understood that both $\boldsymbol{h}_\alpha^p$ and $\boldsymbol{h}_\alpha^b$ can be represented by $\boldsymbol{h}_\alpha$ which is defined by $n_\alpha = \boldsymbol{f}^T\boldsymbol{h}_\alpha$ (as shown in Figure 3).

$$\boldsymbol{h}_\alpha^p = \boldsymbol{h}_\alpha, \qquad \boldsymbol{h}_\alpha^b = \boldsymbol{h}_\alpha l_\alpha \theta_\alpha \tag{15}$$

And from Eq.(3), the yield function for a truss beam model can be expressed in the form

$$\Phi_\alpha\left(\boldsymbol{f}, \theta_\alpha\right) = \frac{\left|\boldsymbol{f}^T\boldsymbol{h}_\alpha\right|}{n_y} - \overline{\tau}_\alpha\left(\theta_\alpha\right) \leq 0 \tag{16}$$

The expression of Eq.(15) and Eq.(16) means that both plastic and buckling component rate are

proportional to the gradient of the yield surface. Namely the associate flow rule is satisfied in this model. It is noted that Eq.(12) is similar to the extended Koiter's form.
For simplicity, assume that the case that member's slenderness ratio is much less. In this case, the assumption that θ_α is approximately equal to θ^p_α is satisfied. Then the yield function of Eq.(16) can be rewritten in the form

$$\Phi_\alpha\left(\boldsymbol{f},\lambda^p_\alpha\right)=\frac{\left|\boldsymbol{f}^T\boldsymbol{h}_\alpha\right|}{n_y}-\bar{\tau}_\alpha\left(\lambda^p_\alpha\right)\le 0,\qquad \bar{\tau}_\alpha\left(\lambda^p_\alpha\right)=\bar{\sigma}_\alpha\left(\lambda^p_\alpha\right)-\frac{n_y l_\alpha\theta^p_\alpha}{4m_p},\qquad \bar{\sigma}_\alpha\left(\lambda^p_\alpha\right)=\sqrt{1+\left(\frac{n_y l_\alpha\theta^p_\alpha}{4m_p}\right)^2} \tag{17}$$

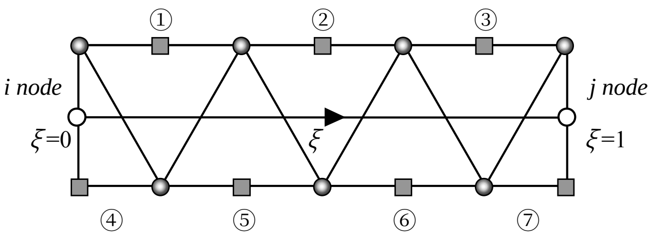
where $\bar{\tau}_\alpha$ is an effective yield stress. Consequently the above assumption enables to treat the elastoplastic buckling problem as the plastic problem with the hardening (softening) specified with Eq.(17-b). And the yield surface becomes to be so-called multi-surface in (M_i, M_j, N) space as Eq.(17-a) must be satisfied at α=1,all. Here we introduce an effective plastic-buckling component $\tilde{\boldsymbol{u}}^{pb}$ which is defined by $\tilde{\boldsymbol{u}}^{pb}=\tilde{\boldsymbol{u}}^p+\tilde{\boldsymbol{u}}^b$ and $\dot{\tilde{\boldsymbol{u}}}^{pb}=\dot{\tilde{\boldsymbol{u}}}^p+\dot{\tilde{\boldsymbol{u}}}^b$. Substituting in the latter equation from Eq.(12) and comparing with Eq.(7) gives

$$\dot{\tilde{\boldsymbol{u}}}^{pb}=\sum_{\alpha=1}^{all}\frac{1}{n_y}\frac{\boldsymbol{f}^T\boldsymbol{h}_\alpha}{\left|\boldsymbol{f}^T\boldsymbol{h}_\alpha\right|}\boldsymbol{h}_\alpha\dot{\lambda}_\alpha\Leftrightarrow\sum_{\alpha\in active}\frac{1}{n_y}\frac{\boldsymbol{f}^T\boldsymbol{h}_\alpha}{\left|\boldsymbol{f}^T\boldsymbol{h}_\alpha\right|}\boldsymbol{h}_\alpha\dot{\lambda}_\alpha\qquad \Theta\ \dot{\lambda}_\beta=0\quad for\quad \Phi_\beta<0 \tag{18}$$

This equation is the plastic flow rule for the present method. Furthermore, the hardening (softening) coefficient can be calculated by

$$\dot{\bar{\tau}}_\alpha=\frac{\partial\bar{\tau}_\alpha}{\partial\theta^p_\alpha}\frac{\partial\theta^p_\alpha}{\partial\lambda^p_\alpha}\dot{\lambda}_\alpha=-\frac{n_\alpha l_\alpha}{16m_p^{\,2}}\frac{1}{\bar{\sigma}_\alpha\left(\theta^p_\alpha\right)^2}\dot{\lambda}_\alpha \tag{19}$$

The yield function, the plastic flow rule and the hardening (softening) property are clarified.



$$n_\alpha=\boldsymbol{f}^T\boldsymbol{h}_\alpha,\qquad \boldsymbol{h}_\alpha=\left\langle\frac{1}{2}\quad \pm\frac{1-\xi_\alpha}{H}\quad \mu\frac{\xi_\alpha}{H}\right\rangle^T$$

■ : gravity of axial force for each chord member

Figure 3. Local coordinate system

3.2 Consistent Tangent Stiffness Matrix

Since $\dot{\Phi}_\alpha=0$ for $\alpha\in active$, substituting in it from Eq.(18) and Eq.(19),

$$\begin{aligned}\dot{\Phi}_\alpha&=\left\{\frac{\partial\Phi_\alpha}{\partial\boldsymbol{f}}\right\}^T\dot{\boldsymbol{f}}-\frac{\partial\bar{\tau}_\alpha}{\partial\lambda^p_\alpha}\dot{\lambda}_\alpha=\frac{1}{n_y}\frac{n_\alpha}{\left|n_\alpha\right|}\boldsymbol{h}_\alpha^{\ T}\boldsymbol{k}^e\left\{\dot{\tilde{\boldsymbol{u}}}-\dot{\tilde{\boldsymbol{u}}}^{pb}\right\}-\frac{\partial\bar{\tau}_\alpha}{\partial\lambda^p_\alpha}\dot{\lambda}_\alpha\\&=\frac{1}{n_y}\frac{n_\alpha}{\left|n_\alpha\right|}\boldsymbol{h}_\alpha^{\ T}\boldsymbol{k}^e\left\{\dot{\tilde{\boldsymbol{u}}}-\sum_{\beta\in active}\frac{1}{n_y}\frac{n_\beta}{\left|n_\beta\right|}\boldsymbol{h}_\beta\dot{\lambda}_\beta\right\}-\frac{\partial\bar{\tau}_\alpha}{\partial\lambda^p_\alpha}\dot{\lambda}_\alpha=0\end{aligned} \tag{20}$$

Then we have the simultaneous equations respect to $\dot{\lambda}_\alpha$ as follows;

$$\sum_{\beta\in active}G_{\alpha\beta}\dot{\lambda}_\beta=\frac{1}{n_y}\frac{n_\alpha}{\left|n_\alpha\right|}\boldsymbol{h}_\alpha^{\ T}\boldsymbol{k}^e\dot{\tilde{\boldsymbol{u}}}\qquad for\quad \alpha\in active \tag{21}$$

where $$G_{\alpha\beta}=\frac{1}{n_y^{\,2}}\frac{n_\alpha}{\left|n_\alpha\right|}\frac{n_\beta}{\left|n_\beta\right|}\boldsymbol{h}_\alpha^{\ T}\boldsymbol{k}^e\boldsymbol{h}_\beta-\frac{n_\alpha l_\alpha}{16m_p^{\,2}\bar{\sigma}_\alpha^{\,2}}\delta_{\alpha\beta}\qquad \delta_{\alpha\beta}\ \text{:kronecker's delta symbol} \tag{22}$$

By solving this simultaneous equation, we can find the plastic consistency parameter $\dot{\lambda}_\alpha$.

$$\dot{\lambda}_\alpha = \frac{1}{n_y}\sum_{\beta\in active} G^{\alpha\beta}\frac{n_\beta}{\left|n_\beta\right|}\boldsymbol{h}_\beta^{\ T}\boldsymbol{k}^e\dot{\tilde{\boldsymbol{u}}} \tag{23}$$

where $G_{\alpha\beta}$ is the inverse of $G^{\beta\alpha}$. And from the nodal force rate the consistent tangent stiffness matrix $\boldsymbol{k}^{epb}$ is given

$$\dot{\boldsymbol{f}} = \left[\boldsymbol{k}^e - \frac{1}{n_y^{\ 2}}\sum_{\alpha\in active}\sum_{\beta\in active}\frac{n_\alpha}{\left|n_\alpha\right|}\frac{n_\beta}{\left|n_\beta\right|}G^{\alpha\beta}\left(\boldsymbol{k}^e\boldsymbol{h}_\alpha\right)\otimes\left(\boldsymbol{k}^e\boldsymbol{h}_\beta\right)\right]\dot{\tilde{\boldsymbol{u}}} \equiv \boldsymbol{k}^{epb}\dot{\tilde{\boldsymbol{u}}} \tag{24}$$

where $\otimes$ signifies tensor product.

3.3 Numerical implementation to calculate nodal force vector

The present calculation method of a nodal force vector belongs to the Return Mapping Algorithm for the Multi-surface yield function (Simo et al.(1988)). The values of ${}^{t}\tilde{\boldsymbol{u}}^{p}$, ${}^{t}\lambda_\alpha^p$ and ${}^{t}\theta_\alpha^p$ in the configuration at t=*t* and ${}^{t+\Delta t}\tilde{\boldsymbol{u}}$ at t=*t+Δt* are assumed to be known.

<u>*Elastic predictor*</u> : Any incremental plastic deformation is frozen; $\Delta\tilde{\boldsymbol{u}}^{pb} = \boldsymbol{0}$, ${}^{trial}\tilde{\boldsymbol{u}}^{pb} = {}^{t}\tilde{\boldsymbol{u}}^{pb}$, $\Delta\lambda_\alpha^p = 0$, ${}^{trial}\lambda_\alpha^p = {}^{t}\lambda_\alpha^p$, ${}^{trial}\theta_\alpha^p = {}^{t}\theta_\alpha^p$. By using these values, trial values can be calculated by

$$ {}^{trial}\boldsymbol{f} = \boldsymbol{k}^e\left({}^{t+\Delta t}\tilde{\boldsymbol{u}} - {}^{trial}\tilde{\boldsymbol{u}}^{pb}\right), \qquad {}^{trial}\overline{\tau}_\alpha = \sqrt{1 + \frac{n_y^{\ 2} l_\alpha^{\ 2}\ {}^{trial}\theta_\alpha^{p\,2}}{16 m_p^{\ 2}}} - \frac{n_y l_\alpha\ {}^{trial}\theta_\alpha^p}{4 m_p} \tag{25}$$

Thus, the trial value of the yield function is given by

$$ {}^{trial}\Phi_\alpha\left({}^{trial}\boldsymbol{f}, {}^{trial}\theta_\alpha^p\right) = \frac{\left|{}^{trial}\boldsymbol{f}^T\boldsymbol{h}_\alpha\right|}{n_y} - {}^{trial}\overline{\tau}_\alpha \tag{26}$$

If ${}^{trial}\Phi_\alpha$ is greater than zero, then the plastic corrector must be execute since the rotary spring is in plastic loading. Otherwise, it is in elastic state including unloading state.

<u>*Plastic corrector*</u> : The stress by the actual plastic deformation is relaxed. The value of the nodal force vector and the effective yield stress at t=*t+Δt* can be rewritten in the form

$$ {}^{t+\Delta t}\boldsymbol{f} = {}^{trial}\boldsymbol{f} - \boldsymbol{k}^e\Delta\tilde{\boldsymbol{u}}^{pb}, \qquad {}^{t+\Delta t}\overline{\tau}_\alpha = \sqrt{1 + \frac{n_y^{\ 2} l_\alpha^{\ 2}\left({}^{trial}\theta_\alpha^p + \Delta\theta_\alpha^p\right)^2}{16 m_p^{\ 2}}} - \frac{n_y l_\alpha\left({}^{trial}\theta_\alpha^p + \Delta\theta_\alpha^p\right)}{4 m_p} \tag{27}$$

Inserting Eq.(27) into the yield function gives the nonlinear simultaneous equations with respect to $\Delta\lambda_\alpha^p(\alpha\in active)$. Then we solve them by Newton method.

$$\begin{aligned} {}^{t+\Delta t}_{(k+1)}\Phi_\alpha &\cong {}^{t+\Delta t}_{(k)}\Phi_\alpha + \sum_{\beta\in active}\frac{\partial\,{}^{t+\Delta t}\Phi_\alpha}{\partial\Delta\lambda_\beta^p}\delta\Delta\lambda_\beta^p \\ &= {}^{t+\Delta t}_{(k)}\Phi_\alpha - \sum_{\beta\in active}\left(\frac{1}{n_y^{\ 2}}\frac{{}^{t+\Delta t}_{(k)}n_\alpha}{\left|{}^{t+\Delta t}_{(k)}n_\alpha\right|}\frac{{}^{t+\Delta t}_{(k)}n_\beta}{\left|{}^{t+\Delta t}_{(k)}n_\beta\right|}\boldsymbol{h}_\alpha^{\ T}\boldsymbol{k}^e\boldsymbol{h}_\beta - \frac{{}^{t+\Delta t}_{(k)}n_\alpha l_\alpha}{16 m_p^{\ 2}\ {}^{t+\Delta t}_{(k)}\overline{\sigma}_\alpha^{\ 2}}\delta_{\alpha\beta}\right)\delta\Delta\lambda_\beta^p \end{aligned} \qquad for \quad \alpha\in active \tag{28}$$

Solving these linear equations and updating each values.

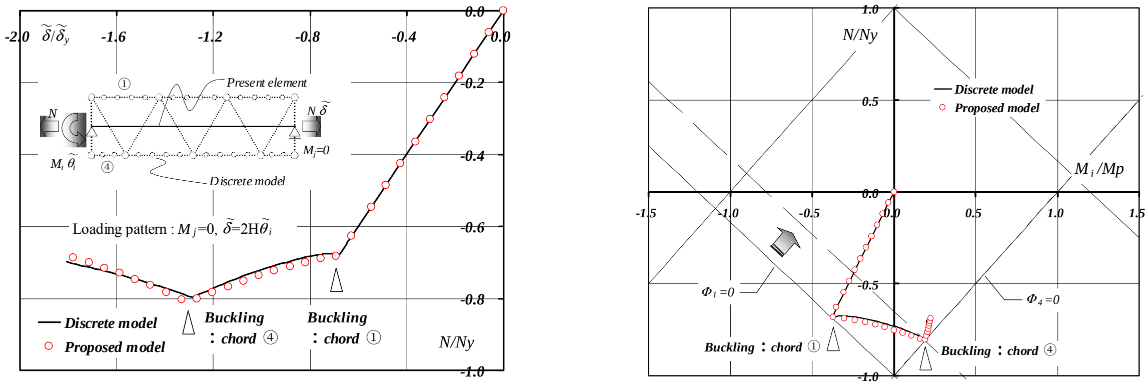
$$ {}^{t+\Delta t}_{(k+1)}\tilde{\boldsymbol{u}}^{pb} = {}^{t+\Delta t}_{(k)}\tilde{\boldsymbol{u}}^{pb} + \sum_{\alpha\in active}\frac{1}{n_y}\frac{{}^{t+\Delta t}_{(k)}n_\alpha}{\left|{}^{t+\Delta t}_{(k)}n_\alpha\right|}\boldsymbol{h}_\alpha\delta\Delta\lambda_\alpha^p, \qquad {}^{t+\Delta t}_{(k+1)}\theta_\alpha^p = {}^{t+\Delta t}_{(k)}\theta_\alpha^p + \frac{{}^{t+\Delta t}_{(k)}n_\alpha}{\left|{}^{t+\Delta t}_{(k)}n_\alpha\right|}\frac{1}{4 m_p\ {}^{t+\Delta t}_{(k)}\overline{\sigma}_\alpha}\delta\Delta\lambda_\alpha^p \tag{29}$$

$$ {}^{t+\Delta t}_{(k+1)}\boldsymbol{f} = \boldsymbol{k}^e\left({}^{t+\Delta t}\tilde{\boldsymbol{u}} - {}^{t+t}_{(k+1)}\tilde{\boldsymbol{u}}^{pb}\right), \qquad {}^{t+\Delta t}_{(k+1)}n_\alpha = {}^{t+\Delta t}_{(k+1)}\boldsymbol{f}^T\boldsymbol{h}_\alpha, \qquad {}^{t+\Delta t}_{(k+1)}\overline{\tau}_\alpha = \sqrt{1 + \frac{n_y^{\ 2} l_\alpha^{\ 2}\ {}^{t+\Delta t}_{(k)}\theta_\alpha^{p\,2}}{16 m_p^{\ 2}}} - \frac{n_y l_\alpha\ {}^{t+\Delta t}_{(k)}\theta_\alpha^p}{4 m_p} \tag{30}$$

where lower index (*k*) signified iteration number. Repeat these calculations until $\left|{}^{t+\Delta t}_{(k+1)}\Phi_\alpha\right| \le Tolerance$.

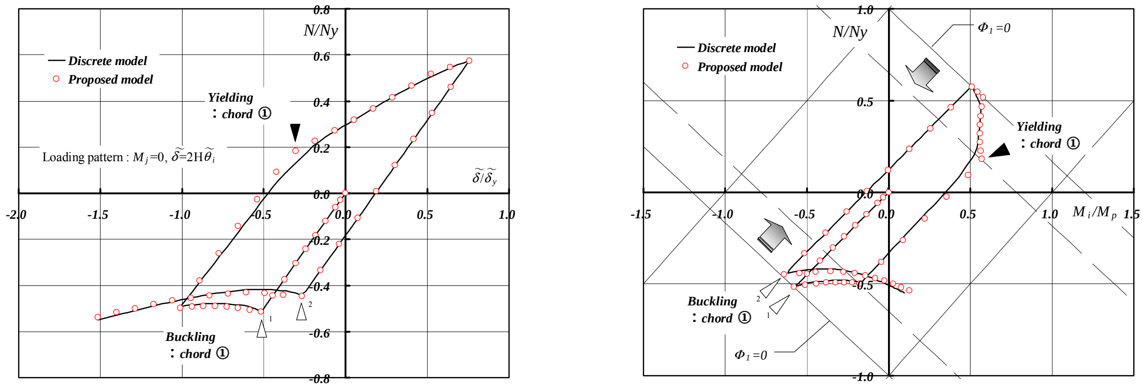
4. NUMERICAL EXAMPLES

Finally we show two numerical examples in Figure 4 and Figure 5 to valid the present method. One example is subjected to monotonic loading and another is subjected to cyclic loading. The present results are calculated with only one element, and another is done with the model of which chords are divided by 10 elastoplastic beam elements. The mechanical properties of analytical model: Truss length; 34.6m, Height; 10m, Slenderness ratio of chord members; 40 (partially 20). The material properties: Young's modulus; 206GPa, Elastic-perfectly plastic material, Yield stress; 235MPa. Two equilibrium paths are close and this means the present method is valid.



(a) Load displacement curve (b) N-M_i space

Figure 4 Numerical results (monotonic loading)



(a) Load displacement curve (b) N-M_i space

Figure 5 Numerical results (cyclic loading)

5. CONCLUSIONS

In this paper, we described the consistent and convenient analytical method to simulate truss beam problems including with the elastoplastic buckling behavior of chord members, and clarified that such problems reduce to the pure elastoplastic problem under the condition that a chord member's slenderness ration is much less. Furthermore the validity of the present method was examined through the numerical examples.

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