SIMPLIFIED PEAK RESPONSE EVALUATION AND DESIGN FOR ELASTO-PLASTICALLY DAMPED STRUCTURES

H. Ito[1)] and K. Kasai[2)]

1) Graduate Student, Department of Built Environment, Tokyo Institute of Technology, Japan
2) Professor, Structural Engineering Research Center, Tokyo Institute of Technology, Japan
ihiroshi@enveng.titech.ac.jp, kasai@serc.titech.ac.jp

Abstract: This paper proposes a pratical theory for peak response evaluation method and a design approach for elasto-plastically damped structure in preliminary seismic design. The proposed theory is based on the single- degree-of-freedom (SDOF) idealization of multistory building structure, and uses the so-called "control performance curve" which simultaneously expresses the seismic performance as a function of stiffness parameter, ductility demand and seismic response spectrum. A rule to convert a SDOF design to a multi- story design and arrangement of damper stiffness over the height of structure is also presented. The accuracy of this method is validated via extensive time history simulations over a wide range of building models.

1. INTRODUCTION

In recent years passive control of building structures by incorporating various energy dissipation devices (dampers) has become common in Japan. In particular, the use of elasto-plastic (EP) damper, such as buckling-restrained brace, for passively-controlled structure have gained widespread practical applications. The EP dampers substantially reduce story drifts and member forces by adding hysteretic damping and stiffness to the primary structure (frame) under earthquake excitation. In preliminary seismic design, however, lack of comprehension of the relationship among response reduction, amount of damper and input ground motion induces an irrational approach, which requires numerous time history simulations.

Objectives of this paper are to propose a practical theory for peak response evaluation method and a design approach for elasto-plastically damped structure in preliminary seismic design, and to verify the accuracy of this method. The proposed theory employs the SDOF idealization of multistory building structure and equivalent linearization technique. A rule to convert a SDOF design to a multi-story design and arrangement of damper stiffness over the height of structure is also presented. The accuracy of this method is validated via time history simulations over a wide range of building models. Basic part of this paper is adopted in "JSSI manual for design and construction of passively-controlled buildings in Japan (2003)".

2. DYNAMIC PROPERTIES OF SDOF EP SYSTEM

2.1 Damper and System

As Figure 1 shows, SDOF model of EP system consists of a mass and two springs which show EP damper and frame connected in a row to the mass. EP damper is modeled as elasto-perfectly-plastic with elastic stiffness K_d and ductility demand μ_d, whereas frame behaves linearly with elastic stiffness K_f (Figure 2(a),(b)). Fundamental vibration period and damping ratio of frame are defined as T_f and h_0. Elastic stiffness K_0, fundamental vibration period T_0 and ductility demand μ of EP system are given by Eq. 1(a)-(c).

$$K_0 = K_f + K_d\,,\quad T_0 = \sqrt{\frac{K_f}{K_0}}\cdot T_f\,,\quad \mu = \mu_d \tag{1a-c}$$

Equivalent linear (secant) stiffness of EP system K_{eq} is

$$K_{eq} = K_f + \frac{K_d}{\mu} = \frac{1+p(\mu-1)}{\mu}\cdot K_0\,,\quad p = \frac{K_f}{K_f + K_d} \tag{2a,b}$$

where p = ratio of post-yield stiffness to elastic stiffness of the system.

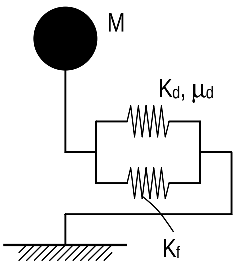

Figure 1
SDOF Model
of EP System

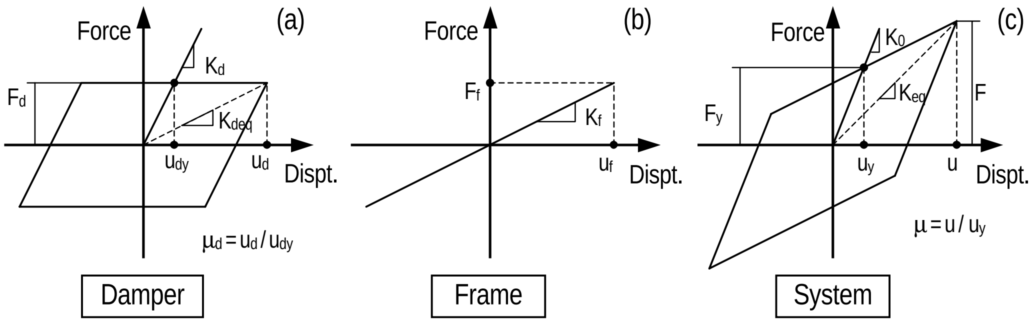

Figure 2 Force – Displacement Characteristic
(a)Damper, (b)Frame and (c)System

2.2 Equivalent Period and Equivalent Damping Ratio of System

According to Eq. 2(a), the equivalent vibration period T_{eq} of system is

$$T_{eq} = \sqrt{\frac{K_f}{K_{eq}}}\cdot T_f = \sqrt{\frac{p\mu}{1+p(\mu-1)}}\cdot T_f \tag{3}$$

The damping ratio of the EP system at ductility demand μ' can be evaluated as the energy dissipated per cycle divided by 4π times the elastic strain energy obtained from secant stiffness. We define the equivalent damping ratio h_{eq} of system as the average of the damping ratio corresponding to ductility factor μ', considering the randomness of earthquake motion as shown in the work by Kasai et al (1998).

$$h_{eq} = h_0 + \frac{1}{\mu}\int_1^{\mu}\frac{2(1-p)(\mu'-1)}{\pi[1+p(\mu'-1)]}d\mu' = h_0 + \frac{2}{\mu\pi p}\ln\left[\frac{1+p(\mu-1)}{\mu^p}\right] \tag{4}$$

3. SYMPLIFIED RESPONSE EVALUATION FOR SDOF EP SYSTEM

3.1 Response Reduction Factor of Displacement and Acceleration

Peak response of the EP system will be obtained from a linear response spectrum using T_{eq} and h_{eq} indicated above. We define S_d, S_{pv}, and S_{pa} as response displacement, response pseudo velocity and response pseudo acceleration spectra, respectively. For the frame, their values are obtained from an expected seismic response spectrum, T_f and h_0. With the response of frame, peak response of the EP system is expressed by considering following two effects due to inserting the damper.

1. The effect of vibration period change (from T_f to T_{eq}) tends to reduce response displacement and increases response acceleration.
2. The effect of hysteretic damping increase (from h_0 to h_{eq}) reduces both response displacement and response acceleration. This effect is represented by damped effect factor D_h, which is an "average" reduction of S_d, S_{pv}, and S_{pa} (Eq. 5).

$$D_h = \sqrt{\frac{1+\alpha h_0}{1+\alpha h_{eq}}} \tag{5}$$

where $\alpha = 25$ (for an ensemble of 31 observed earthquakes from 0.2 to 3 sec of vibration period (Kasai et al., 2003)). Peak responses of the EP system $S_d(T_{eq}, h_{eq})$ and $S_{pa}(T_{eq}, h_{eq})$ normalized to those of the frame $S_d(T_f, h_0)$ and $S_{pa}(T_f, h_0)$ are defined as displacement reduction R_d and pseudo acceleration reduction R_{pa} (for EP system acceleration reduction $R_a = R_{pa}$), respectively. Considered the two effects indicated above, also S_{pv} will be assumed to be period-independent as often assumed for a medium-long period structure. They are given as

$$R_d = D_h \cdot \frac{T_{eq}}{T_f}, \quad R_a = D_h \cdot \frac{T_f}{T_{eq}} \tag{6a,b}$$

Also, for a short period structure, S_{pa} will be assumed to be period-independent, R_d and R_a are given as

$$R_d = D_h \cdot \frac{T_{eq}}{T_f} \cdot \frac{T_{eq}+T_0}{2T_f}, \quad R_a = D_h \cdot \frac{T_f}{T_{eq}} \cdot \frac{T_{eq}+T_0}{2T_f} \tag{7a,b}$$

3.2 Control Performance Curve

The previous equations can clarify the complex interactive effects of stiffness parameter, ductility demand, vibration period, damping and seismic response spectrum on the response reduction of the EP system. Figure 3 shows the curves for drift reduction R_d and acceleration (base shear) reduction R_a of SDOF EP system under a period-independent S_{pv}, and S_{pa}, respectively. The initial damping ratio of frame is $h_0 = 0.02$.

The control performance curves for EP system depend strongly on two parameters: damper stiffness ratio K_d / K_f and ductility demand μ. In Figure 3, the point $K_d / K_f = 0$ gives the frame response $R_d = R_a = 1$. In case of independent-period S_{pv}, to a point, larger K_d / K_f (stiffer damper) leads to smaller drift (R_d) and force (R_a) (Figure 3(a)). Thereafter, the drift continues to decrease, but base shear increases sharply. Also, larger μ (lower yield strength) leads to smaller drift (R_d) and force (R_a). In case of independent-period S_{pa}, larger K_d / K_f and μ lead to smaller drift (R_d) and force (R_a) (Figure 3(b)). As indicated above, the control performance curve clearly shows the trade-off between drift and base shear, and enables to easily obtain the design solution to satisfy the desired response.

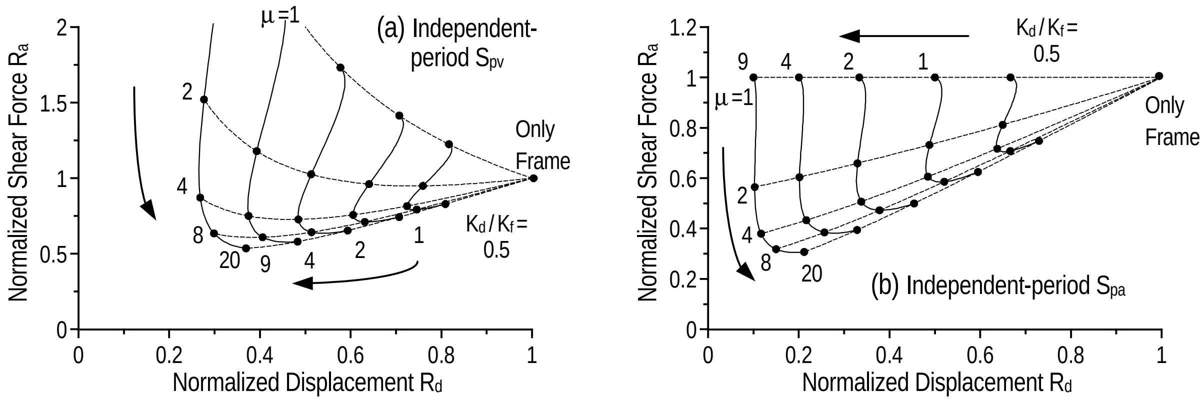

Figure 3 Control Performance Curve

4. DESIGN OF MDOF EP SYSTEM

4.1 Design Conditions of MDOF Frames

Three types of frame are considered: standard type (S-Type), upper-deformed type (U-Type) and lower-deformed type (L-Type). The frames have three different heights: 3, 12, and 24-story. Member stiffness of the frames will be reduced due to incorporating the dampers, fundamental vibration period of them are $T_f = 0.040H$ (12 and 24-story), $0.052H$ (3-story) as shown in table 1. H represents the total height of structure, mass and story height are identical for every story: $m_i = 1.2$ kN·sec^2/cm and $h_i = 4.2$m, respectively. The initial damping ratio of frame is $h_0 = 0.02$.

Consider 12-story frame for example, three types of frame stiffness distribution are shown in Figure 4(a). The frame stiffness K_{fi} at ith-story of S-Type is designed such that story drift becomes uniform under the A_i lateral force distribution (Figure 4(b)). As Figure 4 shows, in U-Type frame, story drift increases at upper stories, whereas in L-Type frame, story drift increases at lower stories. As mentioned above, story stiffness distributions of frames are obtained such that fundamental vibration period of them are $T_f = 2.00$ sec.

Table 1 Fundamental Vibration Period of the 3, 12 and 24-story frames

	3-story	12-story	24-story
T_f (sec)	0.65	2.00	4.00
H (m)	12.6	50.4	100.8
T_f / H	0.052	0.040	0.040

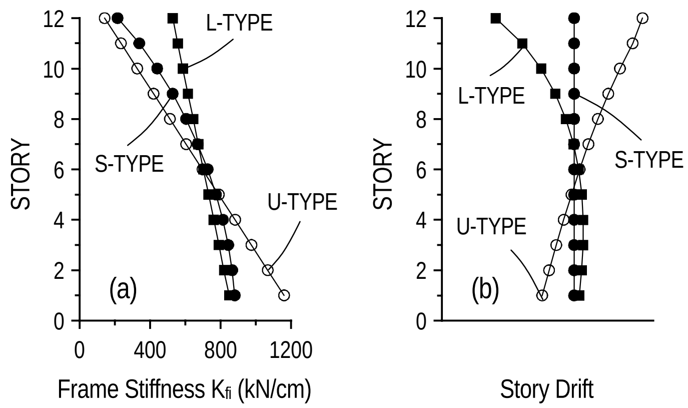

Figure 4 3 Types of 12-Story Frame Stiffness Distributions and Story Drift Distributions

The peak responses S_d, S_{pv}, and S_{pa} of SDOF idealized multi-story frame without damper are obtained from the seismic response spectrum, T_f and h_0. With these response values, displacement u_0 and base shear F_0 of the SDOF frame are given by Eq. 8.

$$u_0 = S_d(T_f, h_0), \quad F_0 = M_{eq} \cdot S_{pa}(T_f, h_0) \tag{8a,b}$$

where $$M_{eq} = \left(\sum_{i=1}^{N} m_i \cdot u_{0i} \right)^2 \Bigg/ \sum_{i=1}^{N} (m_i \cdot u_{0i}{}^2) \tag{9}$$

where M_{eq} = equivalent mass of 1st mode and u_{0i} = deformation shape of frame, which is assumed to be linear over the height of structure regardless of frame type, because desired drift angle distribution of EP system is uniform. Considered that u_0 is displacement of the MDOF frame without damper at equivalent height H_{eq}, drift angle of the SDOF frame θ_f is given by Eq. 10(a).

$$\theta_f = \frac{u_0}{H_{eq}}, \quad H_{eq} = \sum_{i=1}^{N} (m_i \cdot u_{0i} \cdot H_i) \Bigg/ \sum_{i=1}^{N} (m_i \cdot u_{0i}) \tag{10a,b}$$

where H_i = height at ith-story level.

4.2 SDOF EP System Design

For the MDOF frames designed above, SDOF EP systems are designed to meet the performance criteria: yield strength levels of damper corresponds to SDOF EP system ductility demands μ = 2, 4, and 8, and three target drift angles θ_{max} = 1/200, 1/150 and 1/125. Each frame is analyzed for BCJ-L2 artificial ground motion. Firstly, the target displacement reduction factor R_d for each frame is given by Eq. 11.

$$R_d = \frac{\theta_{max}}{\theta_f} \tag{11}$$

Secondly, determine the damper stiffness ratio K_d/K_f at the ductility demand μ to meet the target displacement reduction factor R_d. From response spectrum of BCJ-L2, S_{pv} will be assumed to be period-independent in the range greater than 0.7 sec, S_{pa} will be also assumed to be period-independent in the range of shorter vibration period. Therefore, displacement reduction factors R_d for the SDOF EP system in 12 and 24-story design are obtained by Eq. 6, those of 3-story designs are also obtained by Eq. 7. It is clarified that damped effect factor D_h of BCJ-L2 artificial ground motion is much lower than an 31 ensemble of observed earthquakes in the work by Kasai et al. (2003). In this case, substitute α = 75 (BCJ-L2 artificial ground motion) for Eq. 5.

Considering the indicated above, damper stiffness ratio K_d/K_f to satisfy the target displacement reduction factor R_d can be obtained.

4.3 Conversion to MDOF EP System Design

Considering the change of equivalent stiffness of system K_{eqi} due to yielding of damper under the earthquake excitation, a rule to arrange the damper stiffness K_{di} at ith-story is proposed by Eq. 15 (Kasai et al., 2002). The following constraints are used for the conversion:

1. The equivalent damping, which is ratio of total energy dissipated by damper per cycle divided by 4π times total elastic strain energy obtained from the system secant stiffness, for MDOF EP system becomes the same as that of SDOF EP system.
2. Under the design shear force, the distributions of drift angle and ductility demand of MDOF EP system become uniform, although those of the frame without damper may be non-uniform.
3. Yield drift angle for each story is uniform.

Then, constraint 1 gives

$$\sum_{i=1}^{N}[K_{di}(\mu_i-1)(\theta_i\cdot h_i/\mu_i)^2]\Big/\sum_{i=1}^{N}[(K_{fi}+K_{di}/\mu_i)\cdot\theta_i^2\cdot h_i^2]=K_d(\mu-1)\Big/[(K_f+K_d/\mu)\cdot\mu^2] \tag{12}$$

With constraint 2: drift angle θ_i and ductility demand μ_i at ith-story are $\theta_i=\theta$, $\mu_i=\mu$, respectively, Eq. 12 is revised by Eq. 13.

$$\frac{K_d}{K_f}=\sum_{i=1}^{N}(K_{di}\cdot h_i^2)\Big/\sum_{i=1}^{N}(K_{fi}\cdot h_i^2) \tag{13}$$

where K_d/K_f = damper stiffness ratio obtained from SDOF EP system. Constraint 3 is obviously a necessary and sufficient condition for constraint 2. Also, shear drift angle is a quotient of story shear and stiffness and story height. Thus, from constraint 2

$$Q_i\cdot h_i\Big/[(K_{di}/\mu+K_{fi})\cdot h_i^2]=\sum_{i=1}^{N}(Q_i\cdot h_i)\Big/\sum_{i=1}^{N}(K_{di}\cdot h_i^2/\mu+K_{fi}\cdot h_i^2) \tag{14}$$

where Q_i = the design shear force based on A_i distribution coefficient. Substituting Eq. 13 for Eq. 14, Eq. 15 is obtained.

$$[1+(K_{di}/\mu)/K_{fi}]/[1+(K_d/\mu)/K_f]=\left[Q_i\cdot\sum_{i=1}^{N}(K_{fi}\cdot h_i^2)\right]\Big/\left[K_{fi}\cdot h_i\cdot\sum_{i=1}^{N}(Q_i\cdot h_i)\right] \tag{15}$$

where $K_{di}/\mu=K_{deqi}$: equivalent stiffness of damper at ith-story corresponding to μ. For the frame with uniform story height as considered in this study, Eq. 15 indicates that the equivalent stiffness of system K_{eqi} at ith-story is proportionate to the design shear force Q_i. Consider the condition: 12-story, θ_{max} = 1/150 and μ = 4 for example, distributions of equivalent stiffness of damper K_{deqi} and system K_{eqi} by using the rule mentioned above are shown in Figure 5. As the frame stiffness distribution K_{fi} of S-Type is proportionate to Q_i, the ratio of equivalent stiffness of damper to frame stiffness at ith-story K_{deqi}/K_{fi} evidently becomes uniform value over the height of building. In both U-Type and L-Type frame, K_{deqi}/K_{fi} becomes high value at the story expected large deformation of frame without damper. Whereas, no damper is inserted in the first story for U-Type, and in the top three stories for L-Type.

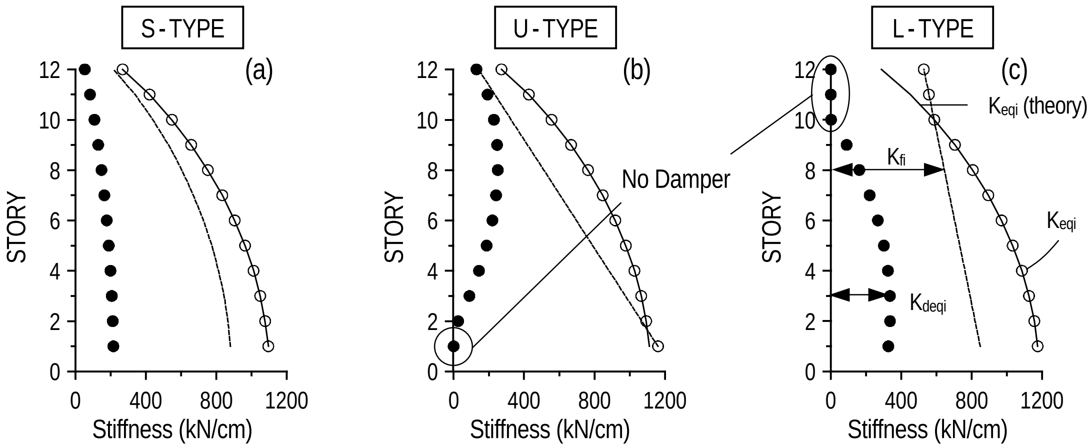

Figure 5 3 Types of Equivalent Stiffness Distributions of Damper and System
(12-Story θ_{max} =1/150, μ=4)

Also, Damper force F_{dyi} at ith-story is given by Eq. 16.

$$F_{dyi} = K_{di} \cdot \Delta u_{yi}\,, \qquad \Delta u_{yi} = \frac{\theta_{max} \cdot h_i}{\mu} \tag{16}$$

5. NUMERICAL RESULTS

Time history simulations were carried out for 81 MDOF EP systems designed above: 3 types of frame, 3 building height, 3 ductility demands, and 3 target drift angle. Simulation models are MDOF shear-bar models as shown in Figure 6. Consider the condition: 3 types of 12-story frames, θ_{max} = 1/150 and μ= 2, 4, and 8 for example, the peak drift angle obtained from time history simulations and design target are shown in Figure 7. As you can see Figure 7, simulation results fairly meet design target due to inserting a sufficient amount of damper. In addition, note that distributions of peak drift angle become uniform regardless of the deformation shape of each frame without damper. Table 2 summarizes the average accuracy of the drift angle for each frame type and building height. "Average" in Table 2 indicates the total average of the ratio of simulation to design target at every story for 9 cases: 3 ductility demands, 3 target drift angle. Compared 3, 12, and 24-story systems, the peak drift angle of the taller building tends to be underestimated. The most likely reason for this issue is that the present approach neglects the contribution of higher modes in evaluating the story drift of MDOF EP system, considering first mode alone is slightly inadequate for 24-story systems.

As a whole, the proposed response evaluation method based on SDOF can provide a good estimation for response of MDOF EP system in preliminary seismic design. It demonstrates that the simple rule to arrange the damper stiffness shown in Eq. 156 can produce the uniform distribution of peak story drift under earthquake excitation.

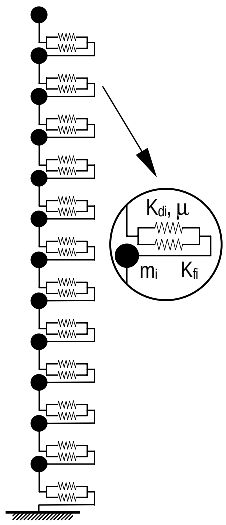

Figure 6 12-DOF Shear-bar Model

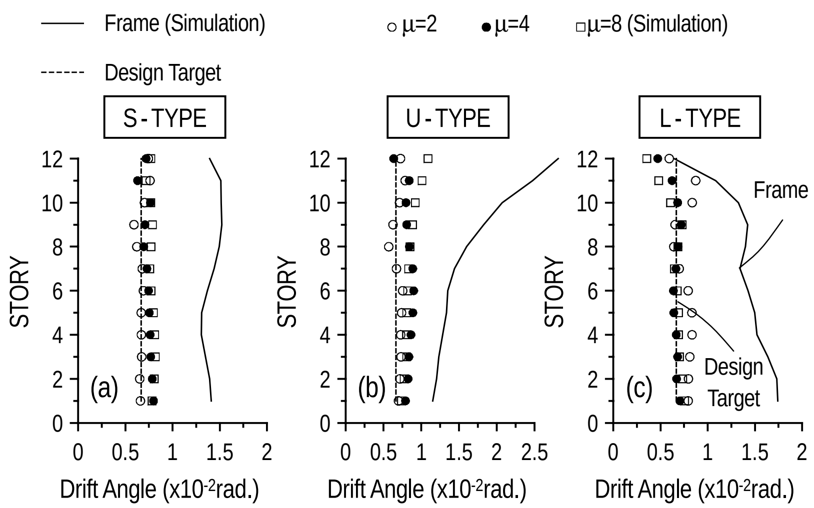

Figure 7 Comparison Simulations with Design Target on Drift Angle (12-Story, θ_{max} =1/150, μ=2, 4, 8)

Table 2 Average Accuracy of Drift Angle

	3-Story			12-Story			24-Story		
	S-Type	U-Type	L-Type	S-Type	U-Type	L-Type	S-Type	U-Type	L-Type
Average	0.890	0.909	0.864	1.102	1.177	0.998	1.229	1.190	1.074
Standard Deviation	(0.102)	(0.145)	(0.112)	(0.105)	(0.167)	(0.150)	(0.176)	(0.196)	(0.235)

6. DESIGN PROCEDURE FOR ELASTO-PLASTICALLY DAMPED STRUCTURES

Characteristics of frame: fundamental vibration period T_f, initial damping h_0, story stiffness distribution K_{fi}, mass distribution m_i, and story height h_i and performance criteria: ductility demand μ, and target drift angle θ_{max} and design response spectrum are all given, design procedure for elasto-plastically damped structure is summarized in sequence of steps below:

1. Obtain the drift angle θ_f and base shear F_0 of SDOF frame without damper from design response spectrum, by evaluating the equivalent height H_{eq} and equivalent mass M_{eq} (Eq. 8-10).
2. Calculate the target displacement reduction factor R_d by Eq. 11.
3. Determine the damper stiffness ratio K_d / K_f at the ductility demand μ to meet the displacement reduction factor R_d by using the control performance curve.
4. Arrange the damper stiffness K_{di} at ith-story by Eq. 15.
5. Calculate the yield deformation Δu_{yi} and strength F_{dyi} of damper at ith-story by Eq. 16.
6. Determine the details of EP dampers as shown in the manual (JSSI, 2003).

7. CONCLUSIONS

This research is aimed toward developing the peak response evaluation method and design approach for elasto-plastically damped structure in preliminary seismic design. The proposed method is based on the SDOF idealization of multi-story building structure, equivalent linearization technique and a rule to convert a SDOF design to a multi-story design. The evaluation of the accuracy of this method for 81MDOF EP systems has led to the following conclusions:

1. The proposed response evaluation method based on SDOF can provide a good estimation for response of MDOF EP system in preliminary seismic design. Design by this approach fairly meets the performance criteria: target drift angle and ductility demand.
2. It demonstrates that the proposed rule to arrange the damper stiffness over the height of structure can produce the uniform distribution of peak story drift under earthquake excitation.

The present approach neglects the contributions of higher modes in evaluating the response of MDOF EP system, considering first mode alone is slightly inadequate for tall buildings such as 24-story systems. It can be further improved by including a sufficient number of modes in evaluating the drift angle.

Acknowledgements:

The Ministry of Education, Science and Culture provided support for this study in the form of Grants-in-Aid for Scientific Research (Research Representative: Hiroshi Ito) and first author receives JSPS fellowship. The author gratefully acknowledges the support.

References:

Japan Society of Seismic Isolation (2003), “JSSI Manual for Design and Construction of Passively-Controlled Buildings in Japan”

Kasai, K., Fu, Y., and Watanabe, A. (1998), “Passive Control Systems for Seismic Damage Mitigation,” *Journal of Structural Engineering*, American Society of Civil Engineers, 124(5), 501-512.

Kasai, K., Ito, H., and Watanabe, A. (2003), “Peak Response Prediction Rule for a SDOF Elasto-plastic System Based on Equivalent Linearization Technique,” *Journal of Structural Engineering*, Architectural Institute of Japan, 571, 53-62

Kasai, K., and Ito, H. (2002), “Stiffness Tuning Method for Design of Elasto-plastically Damped Structures,” *Proceedings of the Passive Control Symposium*, Structural Engineering Research Center, Tokyo Institute of Technology, 273-282