THREE-DIMENSIONAL NON-LINEAR EARTHQUAKE RESPONSE ANALYSIS OF REINFORCED CONCRETE STRUCTURES

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Abstract: In this paper, three-dimensional non-linear earthquake responses of R/C structures were considered with one-mass-system. The restoring force characteristics were modeled based on the theory of plasticity, which was one of the macro models. Two types of the restoring force model were adopted. One was flexural type that could be seen in rigid frame structures, and the other was shear type that could be seen in R/C box wall structures. Parameters of the response analysis were natural period of the system and types of analysis those were three and two dimensional analyses. As a result, comparison between three and two dimensional responses, consideration of vertical response acceleration, and estimation of total energy input were carried out.

1. INTRODUCTION

An earthquake response analysis of building is one of the effective methods in order to estimate the seismic safety. One-directional analysis is popular in the seismic design, and lateral two-directional effects are often considered. However, three-directional analysis is hardly made in the seismic design instead of its necessity. In this paper, three-dimensional non-linear earthquake responses of R/C structures were considered with one-mass-system.

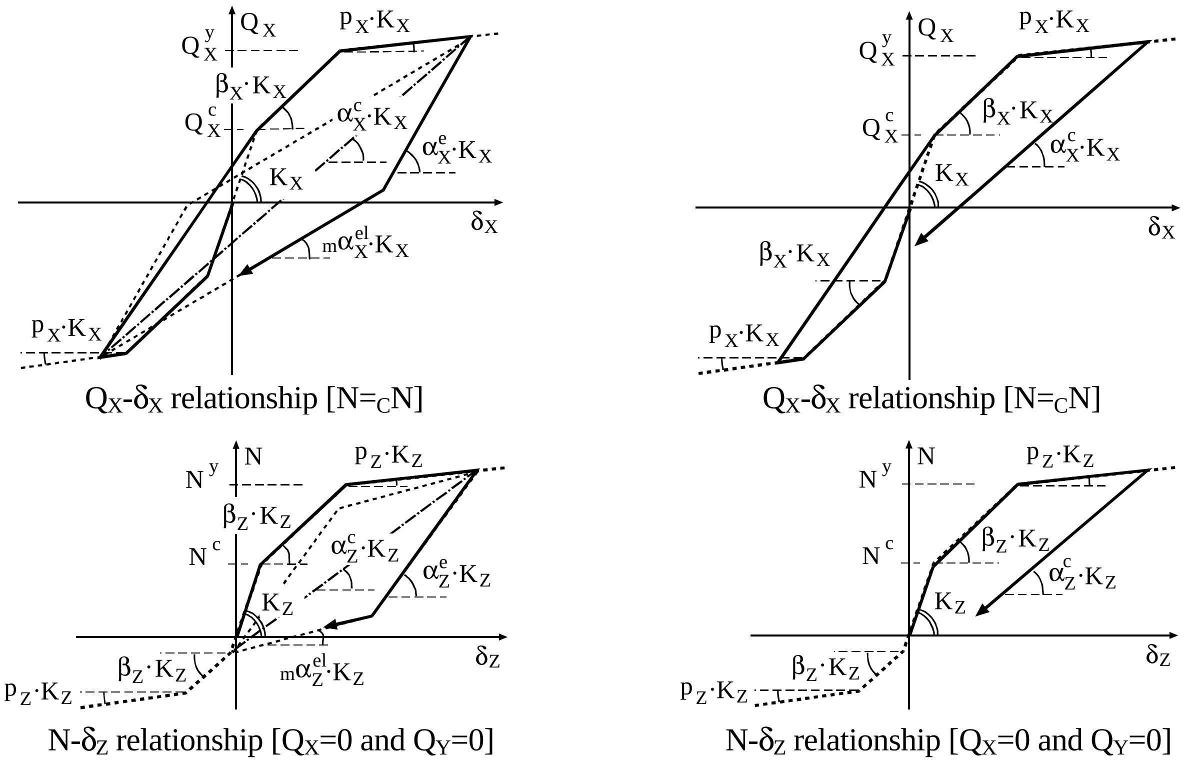
The restoring force characteristics can be represented with skeleton curve and hysterisis loop. It can be said that the characteristics of R/C structures can be classified into two types. One is flexural type that has large area in hysterisis loop, and the other is shear type that has small area in the loop, where the area means energy dissipation. The former one can be seen in rigid frame structures, and the later one can be seen in R/C box wall structures. The restoring force model presented by Nishimura and Takiguchi (2003), which was based on the theory of plasticity so-called analogy model that was one of the macro models, was used for both types with modification of stiffness of hysterisis loop. This model can describe axial deformation behavior on unloading that was one of the problems on analogy models of R/C structures (Takiguchi and Gao 2000).

2. RESTORING FORCE MODEL

2.1 Skeleton Curves

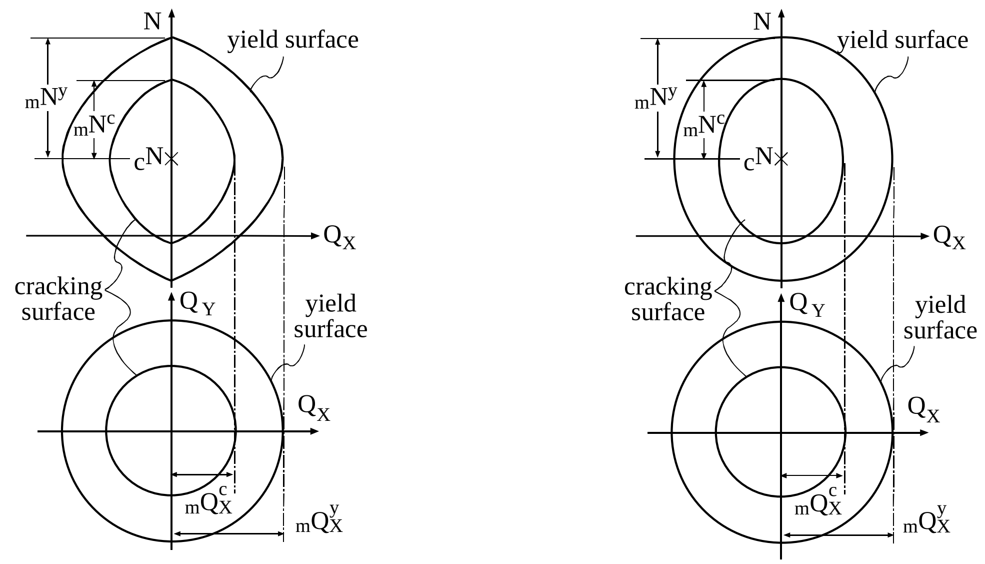
The two types of restoring force model, which are called F-model and S-model in this paper, are represented in tri-linear skeleton curves as shown in Figure 1. The F-model has large area and the

S-model has small area in hysterisis loops. Figure 2 shows cracking surface and yield surface those are corresponded to first and second corners of the skeleton curve, respectively. The F-model has parabola surfaces and the S-model has ellipse, and the cracking surface and the yield surface are expressed as $F^c=0$ and $F^y=0$, respectively. The characteristics of Y-axis are assumed to be the same with X-axis. A force point and a deformation point are expressed as $\{P\}=(Q_X, Q_Y, N)$ and $\{\delta\}=(\delta_X, \delta_Y, \delta_Z)$, respectively, where X and Y axes are lateral axes and Z-axis is vertical axis. The restoring force models of one-direction shown in Figure 1 were expressed when $N={}_CN$. The yield surfaces and the cracking surfaces obey the mixed hardening rule that consist of isotropic hardening and Prager's kinematic hardening rule (Shield and Ziegler 1958).



(a) F-model (b) S-model

Figure 1 Restoring Force Model on One-axis



(a) F-model (b) S-model

Figure 2 Cracking Surface and Yield Surface

2.2 Loading Surface

Loading surface that is corresponded to corner of hysterisis loop of F-model is assumed as $F^l=0$ (Nishimura and Takiguchi 2003). As shown in Figure 3, the loading surfaces are assumed on a virtual plane and Z-axis those are expressed as ${}_bF^l=0$ and ${}_ZF^l=0$, respectively. ${}_bF^l=0$ is represented as circle where $\{{}_bP\}=({}_bQ_X, {}_bQ_Y)^T$, and ${}_ZF^l=0$ decide the range of Z component of a force point.

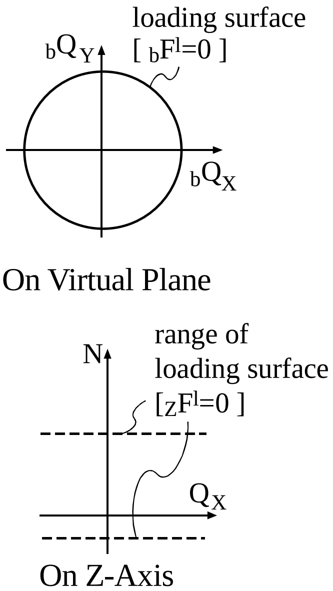


Figure 3 Loading Surface

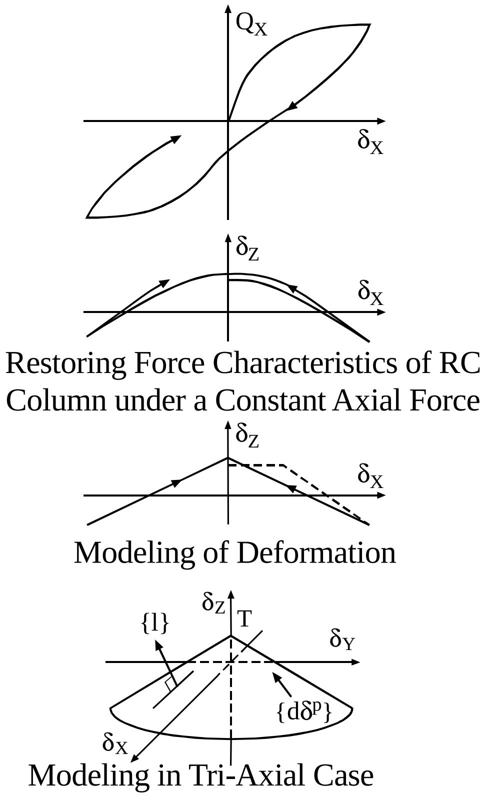


Figure 4 Model of Direction of Plastic Deformation

The force point {P} in tri-axial force space is transformed to the point $\{{}_bP\}$ on the virtual plane with the following equation, where $\{P'\}=({}_bQ_X, {}_bQ_Y, 0)^T$ (Nishimura and Takiguchi 2003).

$$\{dP'\}=k_n\cdot\left[[I]-\frac{\{l\}\cdot\{n\}^T}{\{n\}^T\cdot\{l\}}\right]\cdot\{dP\} \tag{1}$$

[I] is unit matrix, {l} is normal vector of the corn shown in Figure 4, and {n} is a unit vector lies in positive direction of Z-axis. In this paper, k_n is assumed as follows.

$$k_n=1-\left\{(N-{}_CN^c)/{}_mN^c\right\}^2 \text{ for F-model, and } k_n=\sqrt{1-\left\{N-{}_CN^c/{}_mN^c\right\}^2} \text{ for S-model.} \tag{2}$$

A center of the loading surface is also transformed in the same way as the force point.

2.3 Rigidity Degradation

Elastic rigidity degradation of F-model is assumed as shown in Figure 5. The elastic rigidity before yielding is decided as a point direct to the maximum point experienced. The rigidity after yielding is decided according to ratio between maximum yield deformation and initial yield deformation those are expressed as ${}_m\delta^y{}_X$ and ${}_Y\delta_X$ in Figure 5, respectively. The following equation is given to express these rules.

$$\alpha_X^e \cdot K_X = \alpha_X \cdot \left(\frac{{}_m\delta_X^y}{{}_Y\delta_X} \right)^{\gamma} \cdot K_X \tag{3}$$

α_X is the lower value between α^c_X and ${}_{im}Q^y_X/({}_Y\delta_X K_X)$, where ${}_{im}Q^y_X$ is initial value of ${}_mQ^y_X$. ${}_m\delta^y_X$, which initial value is equal to ${}_Y\delta_X$, is assumed to increase as yield surface expands. α^{el}_X shown in Figure 5 is assumed as follows, which is considered axial force effects by using k_n that is equal to Eqn.(2).

$$\alpha_X^{el} = k_n \cdot {}_m\alpha_X^{el}, \text{ where } {}_m\alpha_X^{el} = ({}_mQ_X^c - {}_mQ_X^l) / ({}_mQ_X^c / \alpha_X^c - {}_mQ_X^l / \alpha_X^e). \tag{4}$$

α^e_X of S-model is assumed as follows, which is also considered axial force effects.

$$\alpha_X^e = k_n \cdot \alpha_X^c, \text{ where } k_n \text{ is equal to Eqn.(2).} \tag{5}$$

Rigidities of Y and Z axes are calculated the same as X-axis.

2.4 Top of Corn

A top of the corn shown in Figure 4 is made smooth by parabola to have no corner as shown in Figure 6 because the corner make calculation difficult. In this paper the range of parabola is ${}_m\delta^y_X/50$ in X-direction, and the range in Y- directions are calculated the same as X-direction.

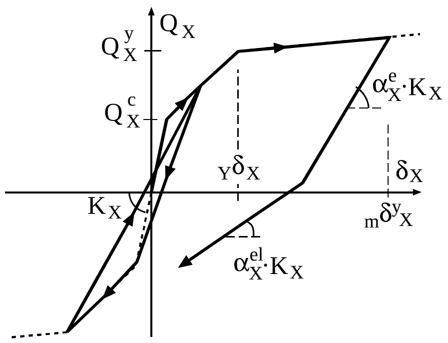


Figure 5 Rigidity Degradation

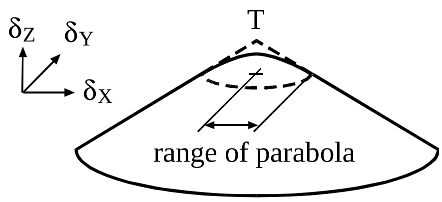


Figure 6 Top of the Corn

3. EARTHQUAKE RESPONSE ANALYSIS

3.1 Numerical Program

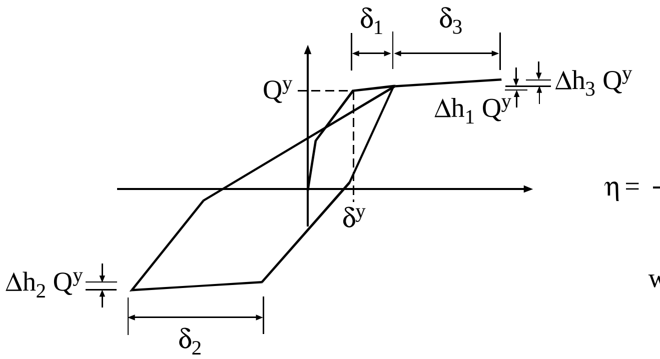
Earthquake responses of R/C structures were examined with the one-mass-system and the two types of restoring force model those were F-model and S-model. Newmark method [β=1/4] was used for the response analysis, and three earthquake ground motions those were Chi Chi 1999, Kobe 1995, and El Centro 1940 were inputted. Constants of the F-model and the S-model were decided based on experimental results of R/C columns (Takiguchi et al. 2001) and R/C box wall structures (Torita et al. 1998), as shown in Table 1. Coefficient of damping was calculated with damping factor and instant stiffness of the system on each step. Parameters of the analysis were natural period ranged from 0.1 to 0.6 and types of analysis those were three and two dimensional analyses. The natural period corresponds to initial elastic stiffness of the model. The restoring model of two-dimensional analysis is the same to the model of three-dimensional analysis except for having no Z-directional components.

A total plastic deformation can be given as shown in Figure 7. A total plastic deformation ratio η

on one-dimension can be estimated by Δh if p, β, and ξ. The equation shown in Figure 7 was adopted in three and two dimensional analyses. In this paper, earthquake response analyses were carried out as η get to 20.0 in F-model and 0.5 in S-model those are decided based on the past experimental study (Takiguchi et al. 2001, and Torita et al. 1998). Δh corresponded to η was calculated, and then analyses were made as Δh get to the calculated values.

Table 1 Constants of the system

	F-model	S-model
β_X (=β_Y=β_Z)	0.27	0.21
p_X (=p_Y=p_Z)	0.001	0.001
Axial force ratio	0.25	0.1
Ratio of yield strength to cracking strength	2.2	3.3
Axial yield strength ratio of tension to compression	0.25	0.25
Axial cracking strength ratio of tension to compression	0.1	0.1
γ shown in Eqn.3	-0.5	-
Damping factor	0.02	0.05
Ratio of vertical natural period to lateral natural period	0.3	0.3



$$\eta = \frac{\text{Total plastic deformation } (=\delta_1+\delta_2+\delta_3)}{\text{Yield deformation } (=\delta^y)} = \frac{\Delta h/p}{\xi + (1-\xi)/\beta},$$

where $\Delta h = \Delta h_1 + \Delta h_2 + \Delta h_3$ and $\xi = \frac{{}_C Q}{{}_Y Q}$

Figure 7 Total Plastic Deformation

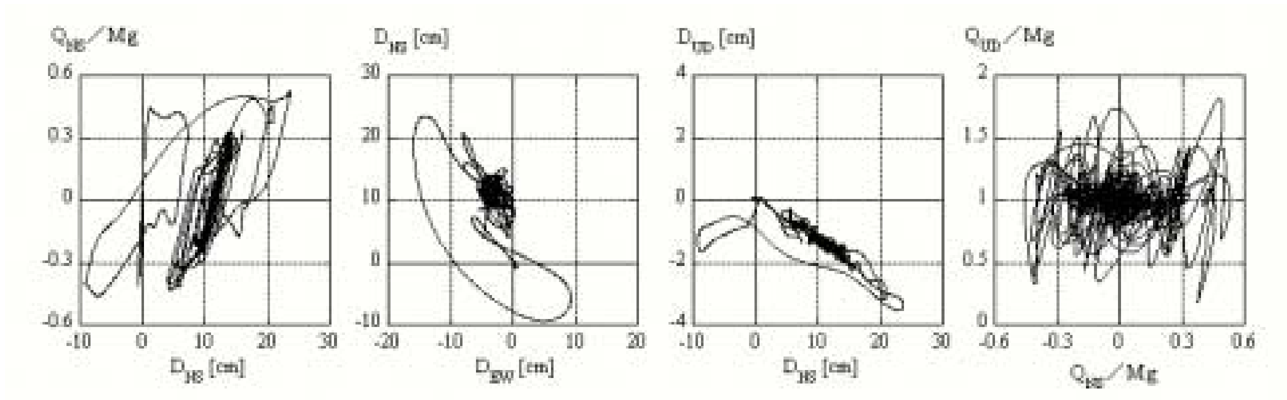
3.2 Numerical Results

Figure 8 shows responses in case of inputting Kobe ground motion. Initial natural periods of the system are 0.2 for F-model and 0.1 for S-model. UD-axis takes compressive side as positive direction. As shown in Figure 8, the restoring force model shown in this paper could represent behaviors on unloading stage, which UD-directional deformation direct to compressive side.

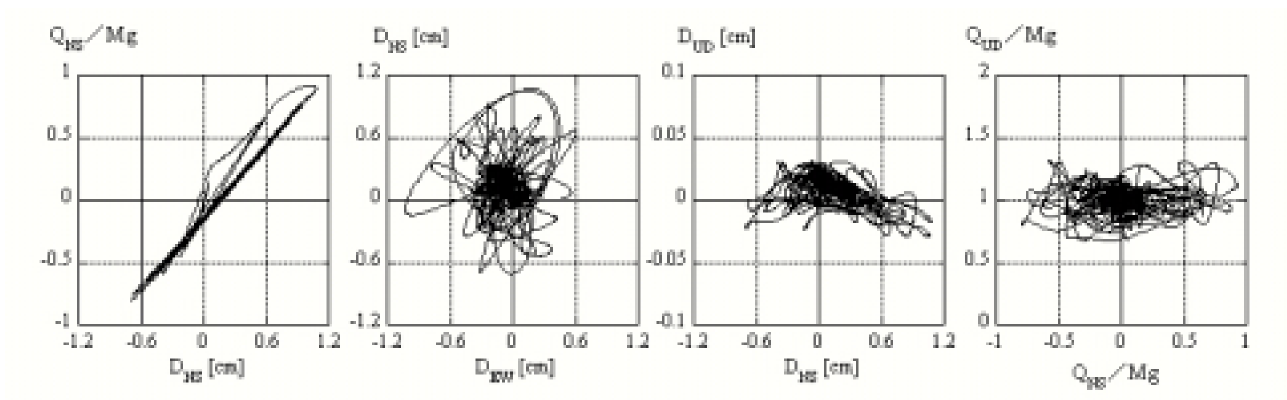
Figure 9 to 12 show numerical results when Chi Chi, Kobe, and El Centro ground motions were inputted. Circle and diamond marks represent the results of F-model and S-model, respectively. Black and White marks represent three and two dimensional analyses, respectively. D, A, A_V, V_E, and T express maximum lateral response deformation, maximum lateral absolute response acceleration, maximum vertical absolute response acceleration, equivalent velocity of total energy input (Akiyama 1985), and initial natural period of the system, respectively. D is equal to a square root of sum of δ_{NS}^2 and δ_{EW}^2. A is calculated in the same way as D. V_E can be given as follows, where E and M are energy input and mass of the system, respectively.

$$V_E = \sqrt{2E/M} \tag{6}$$

D-T, A-T, and V_E-T relationships of three-dimensional analysis almost agree with the relationships of two-dimensional analysis as shown in Figure 9 to 11. These results indicate possibility that it is enough to consider two directional input of earthquake motion when we estimate lateral external force and maximum deformation in seismic design. It is important to know the maximum deformation to judge whether non-structural claddings of buildings can follow the deformation.



(a) F-model with 0.2 sec natural period



(b) S-model with 0.1 sec natural period

Figure 8 Responses inputted Kobe wave

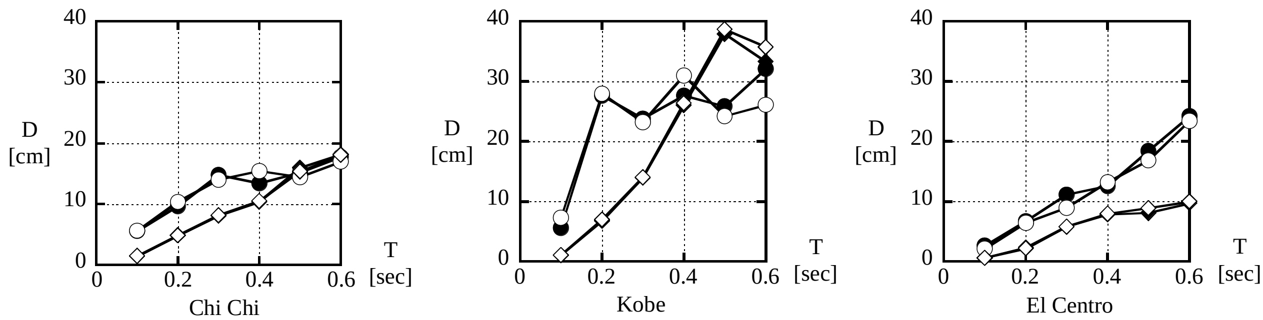


Figure 9 Maximum Lateral response deformation

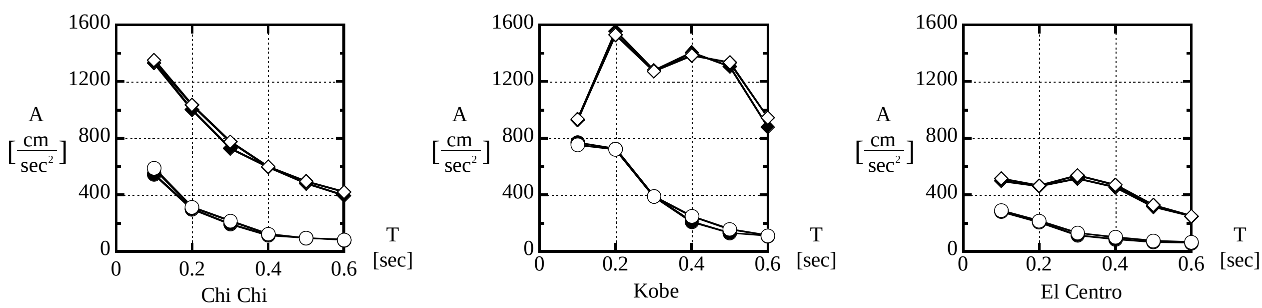


Figure 10 Maximum Lateral Absolute Response Acceleration

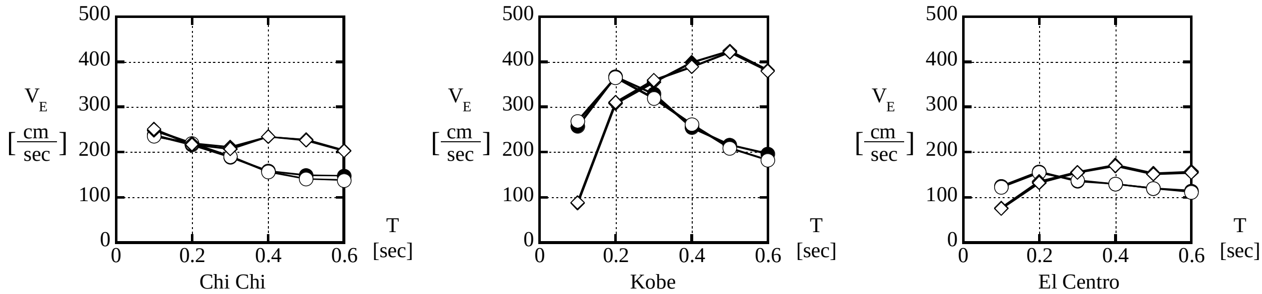


Figure 11 Equivalent Velocity of Total Energy Input

Positive and negative sides of the maximum vertical absolute accelerations are both plotted in Figure 12. As shown in this figure, the values of positive and negative side were almost equal. The biggest values were about 940 cm/sec^2 on the F-model of 0.3 sec natural period inputted Kobe wave, and about 1090 cm/sec^2 on the S-model of 0.5 sec natural period inputted Kobe wave. It can be said that the numerical results of A_V were obtained in the range of 300 to 1100 cm/sec^2. Although this result may not be serious from the viewpoint of collapse of structures, we have to consider this result from the viewpoint of influence on inside of buildings.

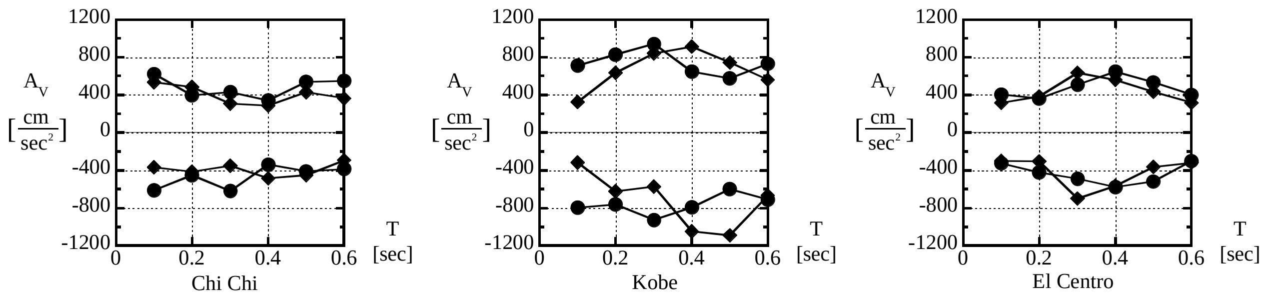


Figure 12 Maximum Vertical Absolute acceleration

3.3 Evaluation of Total Energy Input

An evaluation of the total energy input was considered with a result of elastic response analysis, which damping factor is 0.1 (Akiyama 1985). Figure 13 show V_E-T relationships where a solid curve express a result of three-dimensional elastic response analysis, which damping factor is 0.1. Circle and diamond marks are represented the results of F-model and S-model, respectively. The results were plotted by averaged period of initial period given by initial elastic stiffness and last period given by using last value of α^c_X shown in Figure 1. V_E of F-model and S-model show good agreement with the elastic analysis results. There are some differences between the results of the elastic analysis and non-linear analysis in the range of longer period, however those are safety side and those differences aren't large. Therefore, it can be said that V_E of elastic analysis at averaged natural period have possibility to be able to evaluate V_E of F-model and S-model.

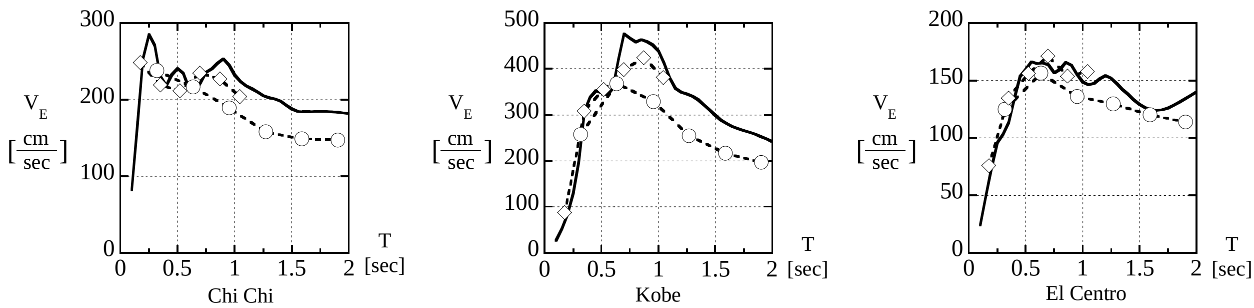


Figure 13 Estimation of Total Energy Input

4. CONCLUSIONS

Three-dimensional non-linear earthquake responses of R/C structures were examined with one-mass system. The restoring force characteristics were modeled based on the theory of plasticity, which was one of the macro models. Two types of the restoring force model were adopted. One was flexural type that had large area in hysterisis loop, and the other was shear type that had small area in the loop. The former one can be seen in rigid frame structures, and the later one can be seen in R/C box wall structures. Parameters of the analysis were natural period ranged from 0.1 to 0.6 and types of analysis those were three and two dimensional analyses. The natural period corresponds to initial elastic stiffness of the model. Chi Chi, Kobe, and El Centro earthquakes were employed for input data. As a result, the following conclusions were found.

1) Maximum lateral response deformation, maximum lateral absolute response acceleration, and total energy input of three-dimensional analysis were almost equal to the responses of lateral two-dimensional analysis. These results indicate possibility that it is enough to consider two directional input of earthquake motion to estimate lateral external force and maximum deformation in seismic design.
2) Maximum vertical absolute response accelerations of three-dimensional analyses were obtained in the range of 300 to 1,100 cm/sec^2. Although this result may not be serious from the viewpoint of collapse of structures, it is necessary to consider this result from the viewpoint of influence on inside of buildings.
3) Total energy input of elastic response analysis with 0.1 damping factor showed good agreement with result of three-dimensional non-linear response at an averaged period that was an average of two periods associated with initial elastic stiffness and last stiffness that connected two maximum points of hysterisis loop diagonally. It can be said the result of elastic response analysis with 0.1 damping factor has possibility to be able to evaluate the total energy input of three-dimensional non-linear earthquake response by using the averaged periods.

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